Balancing for Agent Decision Making through Argumentation

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ABSTRACT

This paper builds bridges between reason-based normative reasoning and formal argumentation, by formalizing Tucker's ethical theory and evaluating the strength of arguments via weighted reasons and tournament semantics. We characterize conditions under which various formulations of permissibility and detachment coincide, and we analyze independence properties that make these equivalences hold. We show that Tucker's input–output "balancing" mechanism, developed for ethics, provides a domain-neutral model of practical reasoning that explains when multiple options are permissible and how to choose among them. As an outlook, we sketch how the mechanism applies to discretionary judicial decision making (e.g., child custody), without claiming that Tucker's ethical theory itself applies to law; instead, the argumentation-based abstraction transfers across normative domains.

KEYWORDS

Artificial Intelligence; Knowledge representation and reasoning; Balancing; Normative reasoning; Discretionary decision making

ACM Reference Format:
Liuwen Yu, Chenyang Cai, Réka Markovich, and Leendert van der Torre. 2026. Balancing for Agent Decision Making through Argumentation. In *Proc. of the 25th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2026), Paphos, Cyprus, May 25 – 29, 2026*, IFAAMAS, 10 pages. https://doi.org/10.65109/VBZK6091

1 INTRODUCTION

Reasoning about what to do is often pictured on a scale: reasons carry weight, and their balance fixes whether an option is permissible, impermissible, or required [20]. Tucker (2025) adds a complementary tournament: options are compared pairwise, and an option counts as permissible only if it is unbeaten. He brings these together by distinguishing justifying from requiring weight and letting pairwise contests track their interaction. In this paper, we use this balancing reasons mechanism because it isolates a formal, domain-neutral pattern of practical reasoning [12, 14, 22, 23, 32, 34].

Our investigation proceeds in two stages.

Part I: Formalization. We translate Tucker's so-called dual scale and tournament semantics into an argumentation framework by argument construction and attack assignment [29, 30], where options are treated as arguments and attacks arise from pairwise competitions on justifying and requiring weights. This formalization allows us to prove equivalences between several formulations of permissibility and detachment and to identify independence conditions under which these equivalences hold. The result generalizes his dual scale computation into a family of semantics for evaluating the strength of arguments and for selecting among permitted options.

Part II: Outlook and application. We show that the same input–output mechanism—evaluating competing options by balancing weighted reasons—extends naturally beyond ethics. As an outlook, we illustrate its use in *discretionary judicial decision making*, bringing examples from child custody cases. Here, judges weigh pros and cons of parents "competing" for custody under the normative constraint of the judge's "duty of care" when deciding and aiming for the "best interest of the child", rather than applying predefined rules, we see a structure that mirrors Tucker's balancing of reasons. We do not claim Tucker's theory applies to law, but its abstract mechanism generalizes across normative domains.

The paper is organized as follows. Section 2 recalls Tucker's key notions. Section 3 formalizes the dynamic-scale model, where permissibility arises from tournament-style comparisons. Section 4 embeds this structure in argumentation theory by treating options as arguments and defining attacks through dual scale detachment. Section 5 refines the framework to address indecision through conflict expansion and preference mechanisms. Section 6 outlines the outlook for applying the reasoning mechanism to discretionary decision making. Section 7 discusses related work, Section 8 discusses future work and concludes.

2 NORMATIVE REASONING

This section highlights key assumptions from Tucker's theory (2025) needed to understand the formal model in Section 3. For analysis of benchmark examples and details on philosophical concepts, motivations, and broader literature on reasons, see Tucker's analysis.

2.1 Ethical Theories for Normative Reasoning

In AI ethics and machine ethics, we are interested in formalizing ethical reasoning, though ethical theories are hard to formalize because they are expressed in ambiguous natural language. In this paper, we use the theory of Tucker (2025), because its claims are

Proc. of the 25th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2026), C. Amato, L. Dennis, V. Mascardi, J. Thangarajah (eds.), May 25 – 29, 2026, Paphos, Cyprus. © 2026 International Foundation for Autonomous Agents and Multiagent Systems (www.ifaamas.org). https://doi.org/10.65109/VBZK6091

about the normative system's functional structure in the following sense: "the normative system takes certain things as inputs (the weights of reasons) and goes from inputs to deontic outputs in a specific way (in virtue of the relative weights of those reasons)." This functional nature of Tucker's work facilitates formalization and experimentation. For example, once the theory is formalized, we can use abstraction, do impact analysis, or study the dynamics of the ethical theory. The issue we are interested in is that once formalized, we can generalize the theory to include other kinds of normative reasoning, such as some types of legal reasoning or rational decision making.

2.2 Tournament Metaphor

We call Tucker's use of "tournament" a metaphor, because it uses the analogy between comparing options and comparing players in a competition: "ϕ is permissible by winning a tournament, a pairwise competition with each alternative." It suggests so-called *comparativism*: "reasons are always differences or comparisons between ϕ and some specified alternative" and so-called *contrastivism*: "for some possible ϕ, the reason for ϕ can vary as you vary the alternative." Tucker discusses the following benchmark example called the Café or Kid Case Problem [36]. "There is a child stuck in a burning building. Your options are Bystander (hang around and watch the events unfold), Save1 (save the child and get severely burned in the process), and Café (keep your promise to meet your friend at the café)." This is modeled as a tournament in which Café wins the competition against Bystander, while the other two competitions are undecided. Reasons may be comparative or contrastive, for example, "the child's death is a reason against Café in the pairwise competition with Save1 but not in the pairwise competition with Bystander."

2.3 Dual Scale Metaphor

In the *Café or Kid* Case, *Save1*, *Bystander*, and *Café* are options. In general, an option "is something I have control over," and privileged options are "those whose deontic status is directly determined by the relative weights of basic reasons." Privileged options are maximal: "they are exhaustive, mutually exclusive, and maximally specific (or maximally specific enough)." Moreover, "basic reasons are always reasons for or against privileged (so maximal) options." The child's death, for instance, is a ground that has weight for *Save1*. In general, a ground "is anything that is a reason in at least one context. Given the conceptual connection between reasons and weight, it follows that a ground is anything that has weight for (or against) some action in at least one context."

There are two functional roles of reasons: *justifying weight (JW)* and *requiring weight (RW)*. A reason's justifying weight for φ is "how hard it pushes φ toward permissibility", while its requiring weight for φ is "how hard it pushes the alternative, $\neg\varphi$, toward impermissibility". Tucker distinguishes reasons by the proportion of their justifying and requiring weight [48]. Self-interested reasons are typically *merely justifying*—"it has justifying but not requiring weight." Altruistic reasons are often *justifying-heavy*—"both justifying and requiring weight, but more of the former." *Balanced reasons* have "exactly as much requiring as justifying weight." *Merely requiring reasons* have "requiring but no justifying weight," and *requiring-heavy reasons* have "both, but more requiring than justifying." For example, avoiding burns is a merely justifying reason.

Building on this distinction, Tucker introduces the *Dual Scale* model to distinguish different types of normative force [48, §0.3, §6.1.2]. One scale determines whether φ is *permissible*: an act φ is permissible iff its justifying weight for φ is not smaller than the requiring weight for its alternative. The other scale determines whether φ is a *commitment* (that is, whether $\neg\varphi$ is impermissible): an act φ is a commitment iff its requiring weight for φ exceeds the justifying weight for its alternative.

Example 2.1 (The Café or Kid *Case).* The weights used in this example are from Tucker's book [48, §9.1], where the comparative force of reasons varies across pairwise competitions. In the comparison between *Bystander* (o_B) and *Save1* (o_{S1}), *Bystander* has $(JW, RW) = (500, 0)$, while *Save1* has $(100, 100)$. When *Café* (o_C) competes with *Bystander*, *Bystander* has $(0, 0)$ and *Café* has $(0, 25)$. Table 1 summarizes these values under Tucker's dual scale model and their deontic status.

Comparison	Permissible		Commitment	
Café vs. Bystander	$25JW_{o_C}$	$0RW_{o_B}$	$0JW_{o_B}$	$25RW_{o_C}$
	verdict: permissible		verdict: commitment	
Café vs. Save1	$525JW_{o_C}$	$100RW_{o_{S1}}$	$100JW_{o_{S1}}$	$25RW_{o_C}$
	verdict: permissible		verdict: not a commitment	
Save1 vs. Bystander	$100JW_{o_{S1}}$	$0RW_{o_B}$	$500JW_{o_B}$	$100RW_{o_{S1}}$
	verdict: permissible		verdict: not a commitment	

Table 1: Dual Scale for Café or Kid [48]

2.4 Computational Aspects

What is striking in Tucker's book is that he not only considers conceptualization, but also computational aspects of tournaments and dual scale competitions. For example, "When there are only two alternatives, the permission and commitment tournaments for ϕ will each involve two competitions (ϕ vs. A_1, ϕ vs. A_2) ... It would be tedious to think through all 20 competitions just to figure out whether ϕ is required." In this paper, we address this issue by independence assumptions, as used, for example, in Bayesian networks and formal argumentation [47].

3 DYNAMIC SCALE DETACHMENT

In this section, we present the formalization of Tucker's theory we introduced in Section 2, including the types of reasons and the independence assumptions. We present two equivalent formalizations, one based on comparative reasons, which facilitates the formal presentation and proofs, and one based on binary choices, which is closer to Tucker's informal presentation, and facilitates the generalization to abstract argumentation in Section 4.All proofs are available online [1].

3.1 Comparative Reasons, Weighting, Weighing and Competitions

In the so-called dynamic scale model, a reason is a ground for an option in contrast to another option.

[1] https://drive.google.com/file/d/18-yUFbBfrBmA4n4yMFvITLJorIfH6NrE/view?usp=sharing

Definition 3.1 (Comparative reasons). Let G and O be two sets called grounds and options, respectively. The set of reasons R_{cr} is a three-place relation $R_{cr} \subseteq G \times O \times O$, where $(g, o_1, o_2) \in R_{cr}$ implies that $o_1 \neq o_2$. We say that the ground g is a reason for option o_1 compared to option o_2.

Each comparative reason is associated with two weights: a justifying weight and a requiring weight.

Definition 3.2 (Weighting for comparative reasons). Let $\mathbb{R}_{\geq 0}$ be the set of nonnegative reals. A weighting function w_{cr} assigns to each comparative reason $(g, o_1, o_2) \in R_{cr}$ two weights w_1 and w_2, such that at least one of w_1 or w_2 is positive. The first weight is called justifying weight, and the second weight is called requiring weight: $jw_{cr}(g, o_1, o_2) = w_1$ and $rw_{cr}(g, o_1, o_2) = w_2$ iff $w_{cr}(g, o_1, o_2) = (w_1, w_2)$.

Pairwise comparisons between options o_1 and o_2 are evaluated using Tucker's dual scale model from a local perspective. o_1 is permissible with respect to o_2 if and only if the sum of justifying weight from reasons for o_1 against o_2 (denoted $sum_{jw}(o_1, o_2)$) is at least as great as the sum of requiring weight from reasons for o_2 against o_1 (denoted $sum_{rw}(o_2, o_1)$). Similarly, o_1 is a commitment with respect to o_2 if and only if $sum_{rw}(o_1, o_2) > sum_{jw}(o_2, o_1)$.

Definition 3.3 (Weighing for comparative reasons). Let scale values $V = \{+, -\}$ be positive and negative. A weighing function d_{cr} assigns to each pair of options o_1 and o_2, and a weighting function w_{cr}, two scale values using addition in the following way: $d_{cr}(o_1, o_2, w_{cr}) = (v_1, v_2)$, where

$sum_{jw}(x, y) = \Sigma_{(g,x,y) \in R_{cr}} jw_{cr}(g, x, y)$
$sum_{rw}(x, y) = \Sigma_{(g,x,y) \in R_{cr}} rw_{cr}(g, x, y)$
$v_1 = tr(sum_{jw}(o_1, o_2) - sum_{rw}(o_2, o_1))$
$v_2 = tr(sum_{jw}(o_2, o_1) - sum_{rw}(o_1, o_2))$

$$tr(x) = + \text{ if } x \geq 0, - \text{ if } x < 0$$

Moreover, when the weighting function w_{cr} is clear, we may also write $d_{cr}(o_1, o_2) = (v_1, v_2)$.

From a global perspective, the dynamic scale is defined in terms of the dual scale model: the commitment scale of o_1 is mirrored by the permission scale of o_2 in a pairwise competition, and vice versa. An option is permitted if and only if it wins its tournament (i.e., it is permissible in every pairwise comparison against each alternative).

Definition 3.4 (Dynamic scale detachment). Let values $V = \{+, -\}$ be positive and negative, or permissible and impermissible. A dynamic detachment function d^*_{cr} assigns values to each option o in O as follows: $d^*_{cr}(o) = +$ if and only if $d_{cr}(o, o', w_{cr}) = (v_1, v_2)$ (where $v_1, v_2 \in V$) and $v_1 = +$ for all $o' \in O$ such that $o' \neq o$; $d^*_{cr}(o) = -$ otherwise.

3.2 Simple Reasons for Binary Choices, Weighting, Weighing and Competitions

Apart from comparative reasons, we also define simple reasons for binary choice. These notions are mathematically equivalent but conceptually distinct.

Definition 3.5 (Simple reasons for binary choices). Let $O_2 = \{\{o_1, o_2\} \subseteq O \mid o_1 \neq o_2\}$ be the set of all unequal pairs of O. The set of simple reasons $R_{sr} : O_2 \rightarrow P(G \times O)$ associates a set of reasons with each pair of options.

Definition 3.6 (Interdefinability). R_{sr} and R_{cr} are interdefinable iff $(g, o_1) \in R_{sr}(\{o_1, o_2\})$ iff $(g, o_1, o_2) \in R_{cr}$.

R_{sr} and R_{cr} are interdefinable by the following functions.

Definition 3.7 (c2s function). The function $c2s$, which maps R_{cr} to R_{sr}, is defined by $c2s(R_{cr}) = \{(g, o) \mid (g, o, o') \in R_{cr}, o \neq o' \in O\}$.

Definition 3.8 (s2c function). The function $s2c$, which maps R_{sr} to R_{cr}, is defined by $s2c(R_{sr}) = \{(g, o_1, o_2) \mid (g, o_1) \in R_{sr}(\{o_1, o_2\})\}$.

Proposition 3.9. *R_{sr} and R_{cr} are interdefinable iff $R_{sr} = c2s(s2c(R_{sr}))$ iff $R_{cr} = s2c(c2s(R_{cr}))$.*

Again, we can express the weighting function w_{sr}, weighing function d_{sr} and dynamic scale detachment d^*_{sr} equivalently in terms of simple reasons for binary choices.

Definition 3.10 (Weighting for simple reasons). Let $\mathbb{R}_{\geq 0}$ be the set of nonnegative reals. For each pair of options $\{o_1, o_2\} \in O_2$, w_{sr} assigns two weights w_1 and w_2 to each simple reason rs in $R_{sr}(\{o_1, o_2\})$, such that at least one of w_1 or w_2 is positive. The first weight is called justifying weight, and the second weight is called requiring weight: $jw_{sr}(\{o_1, o_2\}, rs) = w_1$ and $rw_{sr}(\{o_1, o_2\}, rs) = w_2$ iff $w_{sr}(\{o_1, o_2\}, rs) = (w_1, w_2)$.

Definition 3.11 (Weighing for simple reasons). Let scale values $V = \{+, -\}$ be positive and negative. A weighing function d_{sr} assigns to each pair of options $\{o_1, o_2\} \in O_2$, and a weighting function w_{sr}, two scale values using addition in the following way: $d_{sr}(\{o_1, o_2\}, w_{sr}) = (v_1, v_2)$, where

$sum_{jw}(x, y) = \Sigma_{(g,x) \in R_{sr}(\{x,y\})} jw_{sr}(\{x, y\}, (g, x))$
$sum_{rw}(x, y) = \Sigma_{(g,x) \in R_{sr}(\{x,y\})} rw_{sr}(\{x, y\}, (g, x))$
$v_1 = tr(sum_{jw}(o_1, o_2) - sum_{rw}(o_2, o_1))$
$v_2 = tr(sum_{jw}(o_2, o_1) - sum_{rw}(o_1, o_2))$

$$tr(x) = + \text{ if } x \geq 0, - \text{ if } x < 0$$

Definition 3.12 (Dynamic scale detachment). Let values $V = \{+, -\}$ be positive and negative, or permissible and impermissible. A dynamic detachment function d^*_{sr} assigns values to each option o in O as follows: $d^*_{sr}(o) = +$ if and only if $d_{sr}(\{o, o'\}, w_{sr}) = (v_1, v_2)$ (where $v_1, v_2 \in V$) and $v_1 = +$ for all $\{o, o'\} \in O_2$ such that $o' \neq o$; $d^*_{sr}(o) = -$ otherwise.

The weighting, weighing, and dynamic scale detachment functions defined over the two types of reasons are also equivalent. R_{cr} serves as a global collection of comparative reasons, while R_{sr} maps simple reasons to each local pairwise comparison. Accordingly, we adopt R_{cr} when defining global properties of reasons, and R_{sr} when analyzing local pairwise properties. Sometimes, we use R or w to represent reasons or weighting function when no ambiguity arises.

3.3 Types of Reasons

In what follows, we investigate the five types of reasons, along with the independence assumptions concerning reasons and grounds. These notions can be equivalently expressed using either of the representations R_{cr} and R_{sr}.

Reasons differ in their normative profiles along two dimensions: justifying and requiring normative force.

Definition 3.13. For every pair of options o_1 and o_2, each reason $(g, o_1, o_2) \in R_{cr}$ is one of the following five types. A reason is a *justifying reason* if $rw_{cr}(g, o_1, o_2) = 0$, a *requiring reason* if $jw_{cr}(g, o_1, o_2) = 0$, a *justifying heavy reason* if $jw_{cr}(g, o_1, o_2) > rw_{cr}(g, o_1, o_2)$, *requiring heavy reason* if $rw_{cr}(g, o_1, o_2) > jw_{cr}(g, o_1, o_2)$, and a *balanced* reason if $jw_{cr}(g, o_1, o_2) = rw_{cr}(g, o_1, o_2)$.

To study the properties of the weighting function and its role in argumentation theory, we assume that all reasons are of a fixed type.

Definition 3.14. Some properties may be imposed on w_{cr}:

- **Justifying reason weight** For every pair of options o_1 and o_2, and all $(g, o_1, o_2) \in R_{cr}$, $rw_{cr}(g, o_1, o_2) = 0$.
- **Justifying heavy reason weight** For every pair of options o_1 and o_2, and all $(g, o_1, o_2) \in R_{cr}$, $jw_{cr}(g, o_1, o_2) > rw_{cr}(g, o_1, o_2)$.
- **Balanced reason weight** For every pair of options o_1 and o_2, and all $(g, o_1, o_2) \in R_{cr}$, $jw_{cr}(g, o_1, o_2) = rw_{cr}(g, o_1, o_2)$.
- **Requiring heavy reason weight** For every pair of options o_1 and o_2, and all $(g, o_1, o_2) \in R_{cr}$, $rw_{cr}(g, o_1, o_2) > jw_{cr}(g, o_1, o_2)$.
- **Requiring reason weight** For every pair of options o_1 and o_2, and all $(g, o_1, o_2) \in R_{cr}$, $jw_{cr}(g, o_1, o_2) = 0$.

Example 3.15. Table 1 illustrates the proportion of justifying and requiring weight. When *Save1*(o_{S1}) and *Bystander*(o_B) are being compared. The reason to avoid burns, which supports choosing *Bystander*, is a *merely justifying* reason. The altruistic reason to save the child, which supports *Save1*, is a *balanced* reason.

Certain interdependencies arise among the weighting properties induced by the five types of reasons.

Proposition 3.16. *Let* w_{cr} *be any weighting function, then exactly one of the following holds:* w_{cr} *is a* justifying heavy reason weight *or* w_{cr} *is a* balanced reason weight *or* w_{cr} *is a* requiring heavy reason weight.

Proposition 3.17. *We have that the following holds:* w_{cr} *is a* justifying reason weight $\Longrightarrow$ w_{cr} *is a* justifying heavy reason weight *and* w_{cr} *is a* requiring reason weight $\Longrightarrow$ w_{cr} *is a* requiring heavy reason weight.

3.4 Independence Assumptions

Beyond the five types of reasons characterized by their normative profiles, reasons can also be classified from a global perspective into two categories. A simple reason (g, o) is a global-A-reason if it appears in every comparison involving o. Otherwise, (g, o) is a global-E-reason. It is useful to impose independence assumptions that constrain how global simple reasons are distributed across options and comparisons. Moreover, it provides a way for us to refer to the simple reasons of a ground for an option.

- **Reason independence (RI)** Reasons are tied to individual options. The set of reasons in any pairwise comparison depends only on the reasons associated with the two options.
- **Ground independence (GI)** When options share the same grounds, those shared grounds can be ignored in comparisons. Only distinctive reasons — those that support one option but not the other — contribute to the comparison.
- **Local unique ground (LG)** In any comparison, no ground supports both options, avoiding double-counting or ambiguous reasoning when comparing options pairwise.
- **Global unique ground (GG)** It requires that no ground ever supports more than one option — across all possible comparisons.

Definition 3.18. Some properties may be imposed on R_{sr}:

- **(RI)** There is a set $R_1 \subseteq G \times O$ such that for all $\{o_1, o_2\} \in O_2$, we have $R_{sr}(\{o_1, o_2\}) = R_1 \cap G \times \{o_1, o_2\}$.
- **(GI)** There is a set $R_1 \subseteq G \times O$ such that for all $\{o_1, o_2\} \in O_2$, $R_{sr}(\{o_1, o_2\}) = \{(g, o_1) \in R_1 \mid (g, o_2) \notin R_1\} \cup \{(g, o_2) \in R_1 \mid (g, o_1) \notin R_1\}$.
- **(LG)** For all $\{o_1, o_2\} \in O_2$, $\{g_1 \mid (g_1, o_1) \in R_{sr}(\{o_1, o_2\})\} \cap \{g_2 \mid (g_2, o_2) \in R_{sr}(\{o_1, o_2\})\} = \emptyset$.
- **(GG)** $R_2 = \bigcup_{\{o_1, o_2\} \in O_2} R_{sr}(\{o_1, o_2\})$, for all $\{o_1, o_2\} \in O_2$, $\{g_1 \mid (g_1, o_1) \in R_2\} \cap \{g_2 \mid (g_2, o_2) \in R_2\} = \emptyset$.

The properties defined over R exhibit distinct patterns of (in)depen -dence. Ground independence lies between the strong and weak variants of unique ground and may merit further analysis. Reason independence is a particularly strong property, orthogonal to the others—its satisfaction causes the remaining three notions to collapse into one.

Proposition 3.19. *It holds that* (GG) $\Longrightarrow$ (GI) $\Longrightarrow$ (LG)*, and the rest does not necessarily hold.*

Proposition 3.20. *In the special case of* (RI)*, we have that:* (LG) $\Longleftrightarrow$ (GI) $\Longleftrightarrow$ (GG).

In addition to assuming reason independence, we also adopt weight independence. It states that if a simple reason rs appears in multiple comparisons, then its weight should remain invariant across all such comparisons. This reflects the idea that a reason retains the same normative significance regardless of the options it is compared against.

Definition 3.21 (Weight independence). w_{sr} satisfies the *weight independence (WI)* if and only if the following holds: For every pair of options $\{o_1, o_2\}, \{o_1', o_2'\} \in O_2$, and all $rs \in R_{sr}(\{o_1, o_2\}) \cap R_{sr}(\{o_1', o_2'\})$, $w_{sr}(\{o_1, o_2\}, rs) = w_{sr}(\{o_1', o_2'\}, rs)$.

4 OPTION ARGUMENT FRAMEWORK

In this section, we study the strength of arguments based on the formal model of balancing reasons in Section 3, in the following way. The abstract arguments are options, and the attack assignment is the dual scale comparison. We prove three distinct ways in Tucker's theory, which is embedded as a kind of argumentation: using initial extensions (Theorem 4.6), using reason independence and weight independence (Proposition 4.8), or using particular types of reasons (Proposition 4.16). By varying the argumentation semantics, Tucker's model is generalized to deal with dilemmas in various ways. We also define and prove well-foundedness and coherence properties (Theorem 4.21 and 4.24).

4.1 Option Arguments

In (structured) argumentation, we need to construct arguments, assign an attack relation, and apply semantics. In our setting, we define arguments as options, attacks via dual scale detachment, and extensions as sets of permitted options.

Definition 4.1 (Option argument framework). Let O be a set of options, and w be a weighting function. An *option argument framework* (OAF) is a pair $\mathcal{F} = \langle Ar, att \rangle$ where $Ar = O$ is a finite set called arguments and identified with the set of options, and $att \subseteq Ar \times Ar$ is a binary relation over Ar called attack such that o_1 attacks o_2 iff $d_{cr}(o_1, o_2) = (v, -)$, equivalently, $d_{sr}(\{o_1, o_2\}) = (v, -)$, where $v \in V = \{+, -\}$.

For a set $S \subseteq Ar$ and an option $o \in Ar$, we say that S attacks o if there exists $o' \in S$ such that o' attacks o, o attacks S if there exists $o' \in S$ such that o attacks o', $o^-_{\mathcal{F}} = \{o' \in Ar \mid o' \text{ attacks } o\}$ and $o^+_{\mathcal{F}} = \{o' \in Ar \mid o \text{ attacks } o'\}$.

The initial extension contains all unattacked arguments, meaning that each option within it wins its tournament. Hence, the initial extension always exists and is unique.

Definition 4.2 (Initial extension). Let $\mathcal{F} = \langle Ar, att \rangle$ be an option argument framework. $E \subseteq Ar$ is an initial extension iff it contains all the arguments o for which $o^-_{\mathcal{F}} = \emptyset$.

Proposition 4.3. *Let $\mathcal{F} = \langle Ar, att \rangle$ be an option argument framework. Then there exists a unique initial extension.*

Example 4.4 (Continued from Example 3.15). The option argument framework of Example 3.15 is $\mathcal{F} = \langle Ar, att \rangle$, where $Ar = \{o_C, o_{S1}, o_B\}$, and $att = \{(o_C, o_B)\}$. The unique initial extension $E = \{o_C, o_{S1}\}$.

Because the attack assignment is determined by the permissible relation in dual scale, the initial extension also corresponds to dynamic scale detachment.

Lemma 4.5. *For all pair of options o_1 and o_2, we have: $d_{cr}(o_1, o_2) = (v_1, v_2)$ iff $d_{cr}(o_2, o_1) = (v_2, v_1)$.*

Theorem 4.6. *Let $\mathcal{F} = \langle Ar, att \rangle$ be an option argument framework, $E \subseteq Ar$ be an initial extension. For all $o \in Ar$, $d^*_{cr}(o) = +$ if and only if $o \in E$.*

We examine how the graph structure of an option argument framework may vary under different conditions. As a basic property, the attack relation is irreflexive, as follows directly from the definition 4.1.

Proposition 4.7. *Let $\mathcal{F} = \langle Ar, att \rangle$ be an option argument framework, then att is irreflexive.*

Under the assumptions of reason independence and weight independence, the sum of justifying weight and requiring weight for each option o remain invariant across comparisons with any alternative options o', and we use $sum_{jw}(o, *)$ ($sum_{rw}(o, *)$) to denote the sum of justifying weight (requiring weight). Under these assumptions, the resulting OAF graph structure is constrained and not arbitrary.

Proposition 4.8. *Let R satisfy* (RI)*, w satisfy* (WI)*. An option argument framework under the above conditions is a pair $\mathcal{F} = \langle Ar, att \rangle$. We have the following:*

(1) *If there exists $c \in Ar$ with $c \neq a, b$ such that $(c, b) \in att$ and $(c, a) \notin att$, then $sum_{jw}(a, *) > sum_{jw}(b, *)$.*
(2) *If there exists $c \in Ar$ with $c \neq a, b$ such that $(a, c) \in att$ and $(b, c) \notin att$, then $sum_{rw}(a, *) > sum_{rw}(b, *)$.*

Given an option argument framework, we can apply the result of Proposition 4.8 to construct two partial directed graphs $G_{jw} = (D_{jw}, E_{jw})$ and $G_{rw} = (D_{rw}, E_{rw})$, where nodes represent the total weights of arguments, and edges encode the strict comparison relation > over justifying (or requiring) weights. We use G^*_{jw} (G^*_{rw}) to denote the transitive closure of the G_{jw} (G_{rw}). If there exist cycles in G^*_{jw} or G^*_{rw}, then the corresponding OAF is not allowed.

The graph structure of an option argument framework is influenced not only by independence assumptions but also by the type of weighting function employed. When w is a justifying reason weight, a justifying heavy reason weight, or a balanced reason weight, it prevents the emergence of dilemmas (i.e., mutual attacks between arguments). In contrast, when w is a requiring reason weight or a requiring heavy reason weight, every pair of arguments is in conflict.

Proposition 4.9. *Let $\mathcal{F} = \langle Ar, att \rangle$ be an option argument framework. We have the following:*

(1) *If w is a* justifying reason weight *or a* justifying heavy reason weight *or a* balanced reason weight*, then for every $a, b \in Ar$, $(a, b) \in att$ implies $(b, a) \notin att$.*
(2) *If w is a* requiring reason weight *or a* requiring heavy reason weight*, then for every $a, b \in Ar$, $(a, b) \notin att$ implies $(b, a) \in att$.*

Under the independence assumptions, if w belongs to one of the five types of weights, we find that the attack relation in the option argument framework is transitive.

Proposition 4.10. *Let R satisfy* (RI)*, w satisfy* (WI) *and $\mathcal{F} = \langle Ar, att \rangle$ be an option argument framework under the above conditions. If w is one of the five types of weight, then att satisfies the transitivity.*

Under certain conditions, the attack relation in an option argument framework forms a strict partial order and the initial extension is non-empty.

Proposition 4.11. *Let R satisfy* (RI)*, w be a* justifying reason weight *or a* justifying heavy reason weight *or a* balanced reason weight *satisfying* (WI) *and $\mathcal{F} = \langle Ar, att \rangle$ be a finite option argument framework under the above conditions. We have that att is irreflexive, asymmetric and transitive.*

Proposition 4.12. *Let R satisfy* (RI)*, w be a* justifying reason weight *or a* justifying heavy reason weight *or a* balanced reason weight *satisfying* (WI) *and $\mathcal{F} = \langle Ar, att \rangle$ be a finite option argument framework under the above conditions. The initial extension of $\mathcal{F}$ is non-empty.*

4.2 Dung's Semantics

We introduce Dung's admissibility-based semantics based on the concept of defense, and study their use in dual scale detachment.

Definition 4.13 (Admissible [28]). Let $\langle Ar, att \rangle$ be an AF. $E \subseteq Ar$ is *conflict-free* iff there are no arguments a and b in E such that a attacks b. $E \subseteq Ar$ *defends* c iff for all arguments b attacking c, there is an argument a in E such that a attacks b. $E \subseteq Ar$ is *admissible* iff it is conflict-free and defends all its elements.

For their principle-based analysis, Baroni and Giacomin [7] define semantics as a function from argumentation frameworks to sets of subsets of arguments.

Definition 4.14 (Dung semantics [7]). Dung semantics is a function σ that associates with an argumentation framework $\mathcal{F} = \langle Ar, att \rangle$ a set of subsets of Ar, and the elements of $\sigma(\mathcal{F})$ are called extensions.

Dung distinguishes between several definitions of extension, where each extension is a set of accepted arguments.

Definition 4.15 (Extensions [28]). Let $\langle Ar, att \rangle$ be an AF. $E \subseteq Ar$ is a *complete extension* iff it is admissible and it contains all the arguments it defends. $E \subseteq Ar$ is a *grounded extension* iff it is the minimal complete extension (for set inclusion). $E \subseteq Ar$ is a *preferred extension* iff it is the maximal complete extension (for set inclusion). $E \subseteq Ar$ is a *stable extension* iff it is conflict-free, and it attacks each argument which does not belong to E.

Each kind of extension may be seen as an acceptability semantics that formally rules the argument evaluation process. In this article, we use $\sigma \in \{c, g, p, s\}$ to represent Dung semantics complete, grounded, preferred, stable. We use $\sigma \in \{i\}$ to represent option semantics {initial}. There are notable similarities between the initial extension and the grounded extension.

Proposition 4.16. *Let R satisfy* (RI)*, w is one of the five types of weight satisfying* (WI) *and $\mathcal{F} = \langle Ar, att \rangle$ be an option argument framework. The initial extension of $\mathcal{F}$ is grounded.*

In Dung's theory, a well-founded argumentation framework provides a sufficient condition for the coincidence of grounded, preferred, and stable semantics.

Definition 4.17 (Well-founded AF [28]). An argumentation framework is *well-founded* if and only if there doesn't exist an infinite sequence $a_0, a_1, \ldots, a_n, \ldots$ such that for each i, $(a_{i+1}, a_i) \in att$.

Lemma 4.18. *Every* well-founded *argumentation framework has exactly one complete extension which is grounded, preferred and stable.*

Dung also provides a sufficient condition under which an argumentation framework qualifies as well-founded.

Lemma 4.19. *Let $\mathcal{F} = \langle Ar, att \rangle$ be a finite argumentation framework. $\mathcal{F}$ is* well-founded *if there is no cycle in the directed graph of $\langle Ar, att \rangle$*

The coincidence between Dung's semantics and the option semantics can be generalized under stronger conditions.

Lemma 4.20. *Let R satisfy* (RI)*, w be a* justifying reason weight *or a* justifying heavy reason weight *or a* balanced reason weight *satisfying* (WI) *and $\mathcal{F} = \langle Ar, att \rangle$ be an option argument framework under the above conditions. $\mathcal{F} = \langle Ar, att \rangle$ is* well-founded.

Theorem 4.21. *Let R satisfy* (RI)*, w be a* justifying reason weight *or a* justifying heavy reason weight *or a* balanced reason weight *satisfying* (WI) *and $\mathcal{F} = \langle Ar, att \rangle$ be an option argument framework under the above conditions. $\mathcal{F} = \langle Ar, att \rangle$ has exactly one non-empty complete extension which is grounded, preferred, stable and initial.*

Dung also showed that every stable extension is preferred, and every preferred extension is complete; however, the converse implications do not hold. An argumentation framework is said to be coherent if all of its preferred extensions are also stable. Under certain conditions, the option argument framework can be coherent.

Definition 4.22 (Coherent AF [28]). Let $\mathcal{F} = \langle Ar, att \rangle$ be a finite argumentation framework. $\mathcal{F}$ is *coherent* iff every preferred extension of $\mathcal{F}$ is also stable.

Lemma 4.23. *Let R satisfy* (RI)*, w be a* requiring reason weight *or a* requiring heavy reason weight *satisfying* (WI) *and $\mathcal{F} = \langle Ar, att \rangle$ be a finite option argument framework under the above conditions. Every preferred extension of $\mathcal{F}$ is a singleton set.*

Theorem 4.24. *Let R satisfy* (RI)*, w be a* requiring reason weight *or a* requiring heavy reason weight *satisfying* (WI) *and $\mathcal{F} = \langle Ar, att \rangle$ be a finite option argument framework under the above conditions. We have that $\mathcal{F}$ is* coherent.

5 CHOOSING AMONG PERMISSIBLE OPTIONS

In this section we consider an alternative interpretation of the weights, in which a permitted option is preferred to another permitted option if it is more strongly permitted. We proceed in two steps. First we modify the attack assignment such that two permitted options attack each other. Then we add a preference relation over the arguments, based on the weights, and we compute the defeat relation in the usual way. We prove various theorems about the preference based argumentation theory.

Definition 5.1 (Conflict extended frameworks). Let $\mathcal{F} = \langle Ar, att \rangle$ be an option argument framework. The conflict extended framework of $\mathcal{F}$, denoted as $\mathcal{F}^*$, is represented as follows: $\mathcal{F}^* = \langle Ar^*, att^* \rangle$ where $Ar^* = Ar$, $att^* = att \cup \{(a,a) \mid (a,b) \in att \text{ and } (b,a) \in att\} \cup \{(a,b) \mid (a,b) \notin att \text{ and } (b,a) \notin att\}$.

In the case of conflict extended frameworks, every admissible extension contains at most one option argument.

Proposition 5.2. *Let $\mathcal{F} = \langle Ar, att \rangle$ be a finite option argument framework. Each admissible extension of $\mathcal{F}^*$ has at most one argument.*

Extensions containing at most one argument might appear as a rather unusual borderline phenomenon from a knowledge representation perspective. However, consider the following: in addition to option arguments, the framework may include various other types of arguments. From this multi-sorted perspective, it seems quite natural for there to exist a type of argument where at most one can be accepted. The initial extension derived from an option argument framework is unique but may contain multiple arguments, whereas the extensions obtained from a conflict extended framework contain at most one argument but can be multiple. These two frameworks exhibit interesting connections in their extension outcomes: an option is permitted in the option argument framework if and only if it appears in a preferred extension of the corresponding conflict extended framework.

Lemma 5.3. *Let $\mathcal{F} = \langle Ar, att \rangle$ be an option argument framework, $\mathcal{F}^* = \langle Ar^*, att^* \rangle$ be a conflict extended framework of $\mathcal{F}$ and $E \subseteq Ar$ be an initial extension of $\mathcal{F}$. If $E = \emptyset$, then $\emptyset$ is also the preferred extension of $\mathcal{F}^*$.*

Theorem 5.4. *Let $\mathcal{F} = \langle Ar, att\rangle$ be an option argument framework, $\mathcal{F}^* = \langle Ar^*, att^*\rangle$ be a conflict extended framework of $\mathcal{F}$ and $E \subseteq Ar$ be an initial extension of $\mathcal{F}$. $E = \bigcup_{E_i \in \sigma(\mathcal{F}^*), \sigma\in\{p\}} E_i$.*

Example 5.5 (Continued from Example 4.4). The corresponding conflict extended framework is $\mathcal{F}^* = \langle Ar^*, att^*\rangle$, where $Ar^* = \{o_{S1}, o_C, o_B\}$, and $att^* = \{(o_C, o_B), (o_{S1}, o_B), (o_B, o_{S1}), (o_C, o_{S1}), (o_{S1}, o_C)\}$. $\mathcal{F}^*$ has two preferred extensions, $E_1 = \{o_C\}$ and $E_2 = \{o_{S1}\}$. The union of all preferred extensions of $\mathcal{F}^*$ equals the unique initial extension of $\mathcal{F}$.

Corollary 5.6. *Let $\mathcal{F} = \langle Ar, att\rangle$ be an option argument framework, $\mathcal{F}^* = \langle Ar^*, att^*\rangle$ be a conflict extended framework of $\mathcal{F}$ and $E \subseteq Ar$ be a non-empty initial extension of $\mathcal{F}$. $\mathcal{F}^*$ is coherent.*

Corollary 5.7. *Let $\mathcal{F} = \langle Ar, att\rangle$ be an option argument framework, $\mathcal{F}^* = \langle Ar^*, att^*\rangle$ be a conflict extended framework of $\mathcal{F}$ and $E \subseteq Ar$ be a non-empty initial extension of $\mathcal{F}$. $E = \bigcup_{E_i \in \sigma(\mathcal{F}^*), \sigma\in\{s\}} E_i$*

The conflict extension method enables an equivalence between a single permitted-option extension and multiple extensions. The analysis can be further restricted to the sub-framework of the conflict extended framework induced by the set of permitted options.

Definition 5.8 (Restricted sub-framework of conflict extended framework). Let $\mathcal{F} = \langle Ar, att\rangle$ be an option argument framework, $E \subseteq Ar$ be a non-empty initial extension. $\mathcal{F}^*_{\downarrow E} = \langle Ar^*_{\downarrow E}, att^*_{\downarrow E}\rangle$ is a sub-framework of $\mathcal{F}^*$ restricted to E, defined as follows: $Ar^*_{\downarrow E} = E$, $att^*_{\downarrow E} = att^* \cap E \times E$.

Every $\mathcal{F}^*_{\downarrow E}$ is a symmetric argument framework [19], we have the following properties.

Lemma 5.9. *Let $\mathcal{F} = \langle Ar, att\rangle$ be an option argument framework, $E \subseteq Ar$ be a non-empty initial extension. $\mathcal{F}^*_{\downarrow E}$ is coherent.*

Lemma 5.10. *Let $\mathcal{F} = \langle Ar, att\rangle$ be an option argument framework, $E \subseteq Ar$ be a non-empty initial extension. Every $a \in E$ belongs to one preferred (or equivalently, stable) extension of $\mathcal{F}^*_{\downarrow E}$.*

For any conflict extended framework $\mathcal{F}^*$, we call $\mathcal{F}^*_{\downarrow E}$ its core.

Theorem 5.11. *Let $\mathcal{F} = \langle Ar, att\rangle$ be an option argument framework, $E \subseteq Ar$ be a non-empty initial extension. $\sigma(\mathcal{F}^*) = \sigma(\mathcal{F}^*_{\downarrow E})$, where $\sigma \in \{c, g, p, s\}$.*

To minimize the size of the set of permitted options (i.e., preferred semantics of $\mathcal{F}^*_{\downarrow E}$) as much as possible, we introduce a preference relation $\succeq$ over the permitted options. This reduces the number of extensions, ideally resulting in a unique extension.

Definition 5.12 (Preference relation). Let $\mathcal{F} = \langle Ar, att\rangle$ be an option argument framework, $E \subseteq Ar$ be a non-empty initial extension and $a, b \in E$ be two distinct options. $a \succeq b$ if and only if the following holds: $sum_{jw}(a,b) - sum_{rw}(b,a) \geq sum_{jw}(b,a) - sum_{rw}(a,b)$. We say a is justified preferred to b if and only if: $a \succeq b$ and $sum_{jw}(a,b) > sum_{jw}(b,a)$; a is required preferred than b if and only if: $a \succeq b$ and $sum_{rw}(a,b) > sum_{rw}(b,a)$.

Definition 5.13 (Preference-based conflict extended framework). Let $\mathcal{F} = \langle Ar, att\rangle$ be an option argument framework, $E \subseteq Ar$ be a non-empty initial extension. A preference-based conflict extended framework is a triple $\mathcal{F}_\succeq = \langle Ar_\succeq, att_\succeq, \succeq\rangle$, where $Ar_\succeq = E$, $att_\succeq = \{(a,b) \mid a, b \in E \text{ and } a \succeq b\}$.

Theorem 5.14. *Let $\mathcal{F}_\succeq = \langle Ar_\succeq, att_\succeq, \succeq\rangle$ be a preference-based conflict extended framework. $|p(\mathcal{F}_\succeq)| \leq |p(\mathcal{F}^*)|$.*

However, in the general setting, $\emptyset$ can be a preferred extension of $\mathcal{F}_\succeq$. Under the independence assumptions, this case doesn't hold.

Proposition 5.15. *Let R satisfy* (RI), *w satisfy* (WI), *$\mathcal{F} = \langle Ar, att\rangle$ be an option argument framework, $E \subseteq Ar$ be a non-empty initial extension and $\mathcal{F}_\succeq = \langle Ar_\succeq, att_\succeq, \succeq\rangle$ be a preference-based extended conflict framework. $att_\succeq$ is irreflexive and transitive.*

Proposition 5.16. *Let R satisfy* (RI), *w satisfy* (WI). *Every preferred extension of $\mathcal{F}_\succeq$ is a singleton set.*

6 OUTLOOK: DISCRETIONARY JUDICIAL DECISION MAKING

Ethical considerations have often influenced the evolution of law, and it is unsurprising that abstract ethical reasoning reappears in certain forms of legal reasoning. We apply the abstraction derived from Tucker's framework to *discretionary decision making of judges*, a paradigmatic form of legal reasoning under competing considerations. As described in [24], in the civil-law tradition, judicial reasoning is normally considered as a norm-based derivation, but certain domains—such as child custody—intentionally leave some kind of freedom to the judge, called *discretion*, because no single rule can determine the outcome for all factual variations. The lack of ready-to-apply regulative norms makes the discretionary judicial decision making of course more similar to ethical domains than other types of legal reasoning are. This freedom of the judge, however, is created by law and bounded by the *duty of care* [26], which obliges judges to "examine carefully and impartially all relevant aspects of the individual case and to reason their decision accordingly." In family law, the statute provides only the open-textured aim of securing "the best interests of the child" in child custody cases, so the judge must weigh heterogeneous factual grounds—emotional bond, stability, cooperation, schooling, and others—and assign them relative importance. Discretionary judicial decision making is therefore an exercise in balancing model: it is neither mechanical rule-application nor arbitrary choice, but a structured comparison of diverse reasons within normative limits. Our argumentation-based formalization mirrors this structure precisely: options are represented as arguments, grounds as reasons carrying two comparative weights; pairwise competitions identify the set of permitted alternatives, and, when several remain permissible, a preference relation enables a principled final choice—since the judge must ultimately decide among permissible alternatives.

In the following, we illustrate how the model can be applied to a *simplified* child custody case. The numerical weights used in the example are *illustrative and normative*: they reflect a possible modeling of how different grounds may count for and against each option, rather than any factual measurement. The purpose of the example is thus *explanatory*—to show how the reasoning mechanism operates and how normative justification can be made transparent—rather than descriptive of an actual court decision.

Example 6.1 (Comparative reasons in child custody). In this example, neither of the parents filed for shared custody when filing for divorce (and respective custody). Hence, there are two options for the judge: to give the child custody to the mother (o_M), and to give the child custody to the father (o_F). The following are the relevant grounds (facts) for each option.

g_{bond} : Child's stronger emotional bond with o_F,
g_{school} : Mother's home is in a better school district.

$$R_{cr} = \{(g_{\text{bond}}, o_F, o_M),\ (g_{\text{school}}, o_M, o_F)\}.$$

We continue Example 6.1 to illustrate two kinds of weight of o_M and o_F and how they are being weighed.

Example 6.2 (Continued from Example 6.1). The justifying and requiring weight of the two options are as follows.

(g, o_1, o_2)	jw_{cr}	rw_{cr}
$(g_{\text{bond}}, o_F, o_M)$	7	3
$(g_{\text{school}}, o_M, o_F)$	6	2

Then, we compute and compare the sum of the justifying weight of one option and the sum of the requiring weight of the other: $d_{cr}(o_F, o_M) = (\text{tr}(7-2), \text{tr}(6-3)) = (+,+)$, so both parents are *permissible* relative to each other.

Example 6.3 (Continued from Example 6.2). Assume now, in the continued Example 6.2, one of the parents does ask for shared custody as an option when filing for divorce. We add the option o_S = shared custody, and introduce a new ground g_{co} for co-parenting cooperation: $O' = \{o_F, o_M, o_S\}$.

(g, o_1, o_2)	jw	rw	Comment
$(g_{\text{bond}}, o_F, o_M)$	7	3	bond
$(g_{\text{bond}}, o_F, o_S)$	5	3	bond
$(g_{\text{school}}, o_M, o_F)$	6	2	school
$(g_{\text{school}}, o_M, o_S)$	5	2	school
$(g_{\text{co}}, o_S, o_F)$	1	4	cooperation
$(g_{\text{co}}, o_S, o_M)$	1	4	cooperation

Pairwise dual scale verdicts:

$$d_{cr}(o_F, o_M) = (+,+), \quad d_{cr}(o_M, o_F) = (+,+),$$
$$d_{cr}(o_F, o_S) = (+,-), \quad d_{cr}(o_S, o_F) = (-,+),$$
$$d_{cr}(o_M, o_S) = (+,-), \quad d_{cr}(o_S, o_M) = (-,+).$$

Dynamic detachment:

$$d^*_{cr}(o_F) = +, \qquad d^*_{cr}(o_M) = +, \qquad d^*_{cr}(o_S) = -.$$

Hence, custody with the father (o_F) and custody with the mother (o_M) are both permitted, while shared custody (o_S) is not permitted.

Example 6.4 (Continued from Example 6.3). The option argument framework of Example 6.3 is $\mathcal{F} = \langle Ar, att\rangle$, where $Ar = \{o_F, o_M, o_S\}$, and $att = \{(o_F, o_S), (o_M, o_S)\}$. The initial extension is $E = \{o_F, o_M\}$.

Example 6.5 (Continued from Example 6.4). The corresponding conflict extended framework is $\mathcal{F}^* = \langle Ar^*, att^*\rangle$, where $Ar^* = \{o_M, o_F, o_S\}$, and $att^* = \{(o_F, o_S), (o_M, o_S), (o_F, o_M), (o_M, o_F)\}$.

Example 6.6 (Continued from Example 6.4). From the initial extension $E = \{o_F, o_M\}$ that we have obtained, we have:

$$sum_{jw}(o_F, o_M) = 7, \quad sum_{jw}(o_M, o_F) = 6$$
$$sum_{rw}(o_F, o_M) = 3, \quad sum_{rw}(o_M, o_F) = 2$$

Then we have $o_F \succeq o_M$, o_F is both justified and required preferred than o_M. The preference-based extended conflict framework is $\mathcal{F}_{\succeq} = \langle Ar_{\succeq}, att_{\succeq}, \succeq\rangle$, where $Ar_{\succeq} = \{o_F, o_M\}$, $att_{\succeq} = \{(o_F, o_M)\}$. Thus, the decision should be to give custody to the father.

7 RELATED WORK

There are formal accounts of ethical concepts [40, 41]. There are papers formalizing Tucker's theory, for example by defining "detachment functions" to capture Tucker's dual-scale model [21], by introducing numerical balancing operators [37, 38], and by weighted argumentation [45]. In this paper, we formalize Tucker's ethical theory using formal argumentation by constructing arguments and assigning attacks so as to align *argumentation as balancing* with *argumentation as inference* [44, 52, 53]. Much research has investigated the strength of arguments [3, 42, 43], including extensions of argumentation frameworks such as weighted abstract argumentation [2, 11], preference-based argumentation [35], probabilistic approaches [33], value-based argumentation [5], and gradual [8] and ranking-based semantics [1]. These frameworks typically treat weights as abstract quantities or evaluation scores, whereas our framework derives weights from the normative force of reasons, grounded in normative theory, and assigns a specific normative interpretation. In *AI & Law*, recent work has also framed *judicial discretion* as balancing, e.g., through deontic logic [26] and ASP [25].

8 SUMMARY AND FUTURE WORK

This paper (1) formalizes Tucker's dual-scale and tournament mechanisms through formal argumentation, (2) states reason independence and weight independence conditions, (3) proves equivalences between permissibility and dual scale detachment, plus coincidence results with grounded, preferred, and stable semantics, (4) derives structural properties including irreflexivity, transitivity, well-foundedness, and coherence, (5) extends the framework with a conflict extended framework, restricted preference relation for selecting among permitted options, and (6) applies the formal mechanism to discretionary judicial decision making.

Tucker uses the scale metaphor, drawn from physics, to suggest a mechanical aggregation of normative force. Our paper adopts the tournament metaphor, rooted in multi-agent and game theory. While these metaphors appear incompatible in Tucker's book, formal argumentation might be the bridge for integrating these two metaphors. For the attack-defense paradigm shift in formal argumentation [28], we also have gunfight metaphor [16]: an argument is accepted only if its attacker is not. A future work is to generalize these metaphors to multiagent [4, 6, 46, 51], bipolar [18, 50], higher-order argumentation [10, 15, 17, 31], and value-guided argumentation [49]. Another direction is to explore the dynamics and robustness analysis of normative reasoning, e.g., to examine how conclusions shift with new information or gradual changes in factual parameters. We can also explore how Tucker's notions of enablers, disablers, amplifiers, and attenuators can be integrated into formal argumentation, and how to capture dynamics in this setting [13, 27, 39]. Finally, formalization of normative theories makes it possible to conduct experiments using computational tools, for example, through LogiKEy [9] methodology.

ACKNOWLEDGMENTS

This paper was supported by the Luxembourg National Research Fund (FNR) through Logical Methods for Deontic Explanations (LoDEx; INTER/DFG/23/17415164/LoDEx) and the projects The Epistemology of AI Systems (EAI; C22/SC/17111440), DJ4ME – A DJ for Machine Ethics: the Dialogue Jiminy (O24/18989918/DJ4ME), and Symbolic and Explainable Regulatory AI for Finance Innovation (SERAFIN; C24/19003061/SERAFIN). It was also supported by the University of Luxembourg through the Marie Speyer Excellence Grant supporting Formal Analysis of Discretionary Reasoning (MSE-DISCREASON).

REFERENCES

[1] Leila Amgoud and Jonathan Ben-Naim. 2013. Ranking-based semantics for argumentation frameworks. In *International Conference on Scalable Uncertainty Management*. Springer, 134–147.

[2] Leila Amgoud, Jonathan Ben-Naim, Dragan Doder, and Srdjan Vesic. 2017. Acceptability semantics for weighted argumentation frameworks. In *Twenty-Sixth International Joint Conference on Artificial Intelligence (IJCAI 2017)*. International Joint Conferences on Artificial Intelligence (IJCAI).

[3] Leila Amgoud, Dragan Doder, and Srdjan Vesic. 2022. Evaluation of argument strength in attack graphs: Foundations and semantics. *Artificial Intelligence* 302 (2022), 103607.

[4] Ryuta Arisaka, Jérémie Dauphin, Ken Satoh, and Leendert van der Torre. 2022. Multi-agent Argumentation and Dialogue. *IfCoLog Journal of Logics and Their Applications* 9, 4 (2022), 921–954.

[5] Katie Atkinson and Trevor JM Bench-Capon. 2021. Value-based Argumentation. *IfCoLog Journal of Logics and Their Applications* 8, 6 (2021), 1543–1588.

[6] Edmond Awad, Richard Booth, Fernando Tohmé, and Iyad Rahwan. 2017. Judgement aggregation in multi-agent argumentation. *Journal of Logic and Computation* 27, 1 (2017), 227–259.

[7] Pietro Baroni and Massimiliano Giacomin. 2007. On principle-based evaluation of extension-based argumentation semantics. *Artificial Intelligence* 171, 10-15 (2007), 675–700.

[8] Pietro Baroni, Antonio Rago, and Francesca Toni. 2019. From fine-grained properties to broad principles for gradual argumentation: A principled spectrum. *International Journal of Approximate Reasoning* 105 (2019), 252–286.

[9] Christoph Benzmüller, Xavier Parent, and Leendert van der Torre. 2020. Designing normative theories for ethical and legal reasoning: LogiKEy framework, methodology, and tool support. *Artificial intelligence* 287 (2020), 103348.

[10] Antonis Bikakis, Andrea Cohen, Wolfgang Dvorák, Giorgos Flouris, and Simon Parsons. 2021. Joint attacks and accrual in argumentation frameworks. *Journal of Applied Logics* 8, 6 (2021), 1437–1501.

[11] Stefano Bistarelli, Francesco Santini, et al. 2021. Weighted Argumentation. *Handbook of Formal Argumentation, Volume 2* (2021).

[12] Elizabeth Black and Katie Atkinson. 2009. Dialogues that Account for Different Perspectives in Collaborative Argumentation. In *Proceedings of the 8th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2009)*, Keith Decker, João P. A. Bartholdi Sichman, Carles Sierra, and Cristiano Castelfranchi (Eds.). IFAAMAS, Budapest, Hungary, 867–874.

[13] Richard Booth, Souhila Kaci, Tjitze Rienstra, and Leendert van der Torre. 2013. A logical theory about dynamics in abstract argumentation. In *International Conference on Scalable Uncertainty Management*. Springer, 148–161.

[14] AnneMarie Borg and Floris Bex. 2022. Contrastive Explanations for Argumentation-Based Conclusions: Extended Abstract. In *Proceedings of the 21st International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2022)*, Piotr Faliszewski, Viviana Mascardi, Catherine Pelachaud, and Matthew E. Taylor (Eds.). IFAAMAS, Online, 1551–1553. Extended Abstract.

[15] Martin Caminada, Matthias König, Anna Rapberger, and Markus Ulbricht. 2025. Attack semantics and collective attacks revisited. *Argument & Computation* 16, 2 (2025), 151–211.

[16] Martin WA Caminada, Walter A Carnielli, and Paul E Dunne. 2012. Semi-stable semantics. *Journal of Logic and Computation* 22, 5 (2012), 1207–1254.

[17] Claudette Cayrol, Andrea Cohen, and Marie-Christine Lagasquie-Schiex. 2021. Higher-Order Interactions (Bipolar or not) in Abstract Argumentation: A State of the Art. In *Handbook of Formal Argumentation, Volume 2*, Dov Gabbay, Massimiliano Giacomin, Guillermo R. Simari, and Matthias Thimm (Eds.). College Publications, 15–130.

[18] Claudette Cayrol and Marie-Christine Lagasquie-Schiex. 2005. On the acceptability of arguments in bipolar argumentation frameworks. In *European Conference on Symbolic and Quantitative Approaches to Reasoning and Uncertainty*. Springer, 378–389.

[19] Sylvie Coste-Marquis, Caroline Devred, and Pierre Marquis. 2005. Symmetric argumentation frameworks. In *European Conference on Symbolic and Quantitative Approaches to Reasoning and Uncertainty*. Springer, 317–328.

[20] Jonathan Dancy. 2004. *Ethics without principles*. Clarendon Press.

[21] Vincent de Wit, Aleks Knoks, and Leendert van der Torre. 2025. Dual Scale Detachment. In *European Conference on Logics in Artificial Intelligence*. Springer, 205–219.

[22] Franz Dietrich and Christian List. 2013. A reason-based theory of rational choice. *Nous* 47, 1 (2013), 104–134.

[23] Franz Dietrich and Christian List. 2016. Reason-based choice and context-dependence: An explanatory framework. *Economics & Philosophy* 32, 2 (2016), 175–229.

[24] Josephine Dik and Réka Markovich. 2024. Modeling Judicial Discretion with Nuanced Permissions. In *JURIX*. IOS Press BV, 48–59.

[25] Josephine Dik and Réka Markovich. 2025. Judicial Discretion as Normative Reasoning – Deontic Characterization of Judicial Decision Making with Answer Set Programming. In *Proceedings of the 20th International Conference on AI and Law*, Juliano Maranhao (Ed.). ACM, Chicago, 258–267.

[26] Josephine Dik and Réka Markovich. 2025. When Judges Go Wrong: Modeling Discretion and the Duty of Care. In *Deontic Logic and Normative Systems*. College Publications, 399–400.

[27] Sylvie Doutre and Jean-Guy Mailly. 2018. Constraints and changes: A survey of abstract argumentation dynamics. *Argument & Computation* 9, 3 (2018), 223–248.

[28] Phan Minh Dung. 1995. On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. *Artificial Intelligence* 77, 2 (1995), 321–357.

[29] Phan Minh Dung. 2016. An axiomatic analysis of structured argumentation with priorities. *Artificial Intelligence* 231 (2016), 107–150. https://doi.org/10.1016/j.artint.2015.10.005

[30] Phan Minh Dung and Phan Minh Thang. 2018. Fundamental properties of attack relations in structured argumentation with priorities. *Artificial Intelligence* 255 (2018), 1–42. https://doi.org/10.1016/j.artint.2017.11.002

[31] Wolfgang Dvořák, Jorge Fandinno, and Stefan Woltran. 2019. On the expressive power of collective attacks. *Argument & Computation* 10, 2 (2019), 191–230.

[32] Débora C. Engelmann, Alison R. Panisson, Renata Vieira, Jomi Fred Hübner, Viviana Mascardi, and Rafael H. Bordini. 2023. MAIDS — A Framework for the Development of Multi-Agent Intentional Dialogue Systems. In *Proceedings of the 22nd International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2023)*, Alessandro Ricci, William Yeoh, Noa Agmon, and Bo An (Eds.). IFAAMAS, London, United Kingdom, 1209–1217.

[33] Anthony Hunter, Sylwia Polberg, Nico Potyka, Tjitze Rienstra, and Matthias Thimm. 2021. Probabilistic argumentation: A survey. *Handbook of Formal Argumentation* 2 (2021), 397–441.

[34] Junqi Jiang, Francesco Leofante, Antonio Rago, and Francesca Toni. 2024. Recourse under Model Multiplicity via Argumentative Ensembling. In *Proceedings of the 23rd International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2024)*, Natasha Alechina, Virginia Dignum, Mehdi Dastani, and Jaime S. Sichman (Eds.). IFAAMAS, Auckland, New Zealand, 954–963.

[35] Souhila Kaci, Leendert van der Torre, Srdjan Vesic, and Serena Villata. 2021. Preference in Abstract Argumentation. In *Handbook of Formal Argumentation, Volume 2*, Dov Gabbay, Massimiliano Giacomin, Guillermo R. Simari, and Matthias Thimm (Eds.). College Publications, 211–248.

[36] Frances Myrna Kamm. 2001. *Morality, mortality: Volume II: Rights, duties, and status*. Vol. 2. Oxford University Press.

[37] Aleks KNOKS, Muyun SHAO, Leon VAN DER TORRE, Vincent DE WIT, and Liuwen YU. 2024. A Principle-based Analysis for Numerical Balancing. In *Logics for New-Generation Artificial Intelligence (LNGAI2024)*. College Publications, United Kingdom.

[38] Aleks Knoks and Leon VAN DER TORRE. 2023. Reason-based detachment. In *Logics for New-Generation Artificial Intelligence (LNGAI2023)*. College Publications, London, United Kingdom.

[39] Beishui Liao, Li Jin, and Robert C. Koons. 2011. Dynamics of argumentation systems: A division-based method. *Artificial Intelligence* 175, 11 (2011), 1790–1814. https://doi.org/10.1016/j.artint.2011.03.006

[40] Paul McNamara. 1996. Doing well enough: Toward a logic for common-sense morality. *Studia logica* 57 (1996), 167–192.

[41] Robert Mullins. 2021. Formalizing reasons, oughts, and requirements. *Ergo an Open Access Journal of Philosophy* 7 (2021).

[42] Pere Pardo, Liuwen Yu, Chen Chen, and Leendert van der Torre. 2025. Weakest link, prioritized default logic and principles in argumentation. *Journal of Logic and Computation* 35, 4 (2025), exaf007.

[43] Henry Prakken. 2024. An abstract and structured account of dialectical argument strength. *Artificial Intelligence* 335 (2024), 104193.

[44] Tjitze Rienstra, Leon VAN DER TORRE, and Liuwen Yu. 2025. Reasoning alignment for Agentic AI: Argumentation, belief revision, and dialogue. *Journal of Applied Logics-IfCoLoG Journal of Logics and their Applications* 12, 6 (2025).

[45] David Streit, Vincent de Wit, and Aleks Knoks. 2023. Reasons in Weighted Argumentation. In *Logic, Rationality, and Interaction: 9th International Workshop,*

LORI 2023, Jinan, China, October 26–29, 2023, Proceedings. Springer Nature.

[46] Matthias Thimm. 2014. Strategic argumentation in multi-agent systems. *KI-Künstliche Intelligenz* 28 (2014), 159–168.

[47] Matthias Thimm, Pietro Baroni, Massimiliano Giacomin, and Paolo Vicig. 2017. Probabilities on extensions in abstract argumentation. In *International Workshop on Theories and Applications of Formal Argumentation*. Springer, 102–119.

[48] Chris Tucker. 2025. *The Weight of Reasons: A Framework for Ethics*. Oxford University Press.

[49] Bart Verheij. 2016. Formalizing value-guided argumentation for ethical systems design. *Artificial Intelligence and Law* 24 (2016), 387–407.

[50] Liuwen Yu, Caren Al Anaissy, Srdjan Vesic, Xu Li, and Leendert van der Torre. 2023. A Principle-Based Analysis of Bipolar Argumentation Semantics. In *European Conference on Logics in Artificial Intelligence*. Springer, 209–224.

[51] Liuwen Yu, Dongheng Chen, Lisha Qiao, Yiqi Shen, and Leendert van der Torre. 2021. A principle-based analysis of abstract agent argumentation semantics. In *Proceedings of the International Conference on Principles of Knowledge Representation and Reasoning*, Vol. 18. 629–639.

[52] Liuwen Yu and Leendert van der Torre. 2025. The A-BDI metamodel for human-level AI: Argumentation as balancing, dialogue and inference. In *International Conference on Logic and Argumentation*. Springer, 361–379.

[53] Liuwen Yu, Leendert van der Torre, and Réka Markovich. 2024. Thirteen Challenges in Formal and Computational Argumentation. In *Handbook of Formal Argumentation, Volume 3*, Dov Gabbay, Gabriele Kern-Isberner, Guillermo R. Simari, and Matthias Thimm (Eds.). College Publications, 890–976.