A Principle-based Approach to Bipolar Argumentation

Liuwen Yu
University of Luxembourg, Luxembourg, University of Bologna, Italy, University of Turin, Italy
Leendert van der Torre
University of Luxembourg, Luxembourg, Zhejiang University, China

Abstract

Support relations among arguments can induce various kinds of indirect attacks corresponding to deductive, necessary or evidential interpretations. These different kinds of indirect attacks have been used in meta-argumentation, to define reductions of bipolar argumentation frameworks to traditional Dung argumentation frameworks, and to define constraints on the extensions of bipolar argumentation frameworks. In this paper, we give a complete analysis of twenty eight bipolar argumentation framework semantics and fourteen principles. Twenty four of these semantics are for deductive and necessary support and defined using a reduction, and four other semantics are defined directly. We consider five principles directly corresponding to the different kinds of indirect attack, three basic principles concerning conflict-freeness and the closure of extensions under support, three dynamic principles, a generalized directionality principle, and two supported argument principles. We show that two principles can be used to distinguish all reductions, and that some principles do not distinguish any reductions. Our results can be used directly to obtain a better understanding of the different kinds of support, to choose an argumentation semantics for a particular application, and to guide the search for new argumentation semantics of bipolar argumentation frameworks. Indirectly they may be useful for the search for a structured theory of support, and the design of algorithms for bipolar argumentation.

keywords: Abstract argumentation, support, principle-based approach, bipolar argumentation framework

Introduction

In his requirements analysis for formal argumentation, Gordon (2018) proposes the following definition covering more clearly argumentation in deliberation as well as persuasion dialogues: "Argumentation is a rational process, typically in dialogues, for making and justifying decisions of various kinds of issues, in which arguments pro and con alternative resolutions of the issues (options or positions) are put forward, evaluated, resolved and balanced." At an abstract level, it seems that these pro and con arguments can be represented more easily in so-called bipolar argumentation frameworks (Cayrol and Lagasquie-Schiex 2005; 2009; 2010; 2013) containing besides attack also a support relation among arguments.

The concept of support has attracted quite some attention in the formal argumentation literature, maybe because it remains controversial how to use support relations to compute extensions. Most studies distinguish deductive support, necessary support and evidential support. Deductive support (Boella et al. 2010) captures the intuition that if *a* supports *b*, then the acceptance of *a* implies the acceptance of *b*, and as a consequence the non-acceptance of *b* implies the non-acceptance of *a*. Evidential support (Besnard and others 2008; Oren, Luck, and Reed 2010) distinguishes prima-facie from standard arguments, where prima-facie arguments do not require any support from other arguments to stand, while standard arguments must be supported by at least one prima-facie argument. Necessary support (Nouioua and Risch 2010) captures the intuition that if *a* supports *b*, then the acceptance of *a* is necessary to get the acceptance of *b*, or equivalently the acceptance of *b* implies the acceptance of *a*.

Despite this diversity, the study of support in abstract argumentation seems to agree on the following three points.

Relation support and attack The role of support among arguments has been often defined as subordinate to attack, in the sense that in deductive and necessary support, if there are no attacks then there is no effect of support. On the contrary, in the evidential approach, without support, there is no accepted argument even if there is no attack.

Diversity of support Different interpretations for the notion of support can be distinguished, such as deductive (Boella et al. 2010), necessary (Nouioua and Risch 2011; Nouioua 2013) and evidential support (Besnard and others 2008; Oren, Luck, and Reed 2010; Polberg and Oren 2014).

Structuring support Whereas attack has been further structured into rebutting attack, undermining attack and undercutting attack, the different kinds of support have not led yet to a structured argumentation theory for bipolar argumentation frameworks.

The picture that emerges from the literature is that the notion of support is much more diverse than the notion of attack. Whereas there is a general agreement in the formal argumentation community how to interpret attack, even when different kinds of semantics have been defined, there is much less consensus on the interpretation of support. Moreover, it seems that each variant of support can be used for different applications.

This paper contributes to a further understanding of the concept of support using a principle-based analysis. Some of the fourteen principles we study in this paper turn out to discriminate the various reductions based semantics of bipolar argumentation frameworks, and they can therefore be used to choose one semantics over another. Some other principles always hold, or never, and can therefore guide the search for new semantics of bipolar argumentation frameworks.

Principles and axioms can be used in many ways. Often, they conceptualize the behavior of a system at a higher level of abstraction. Moreover, in absence of a standard approach, principles can be used as a guideline for choosing the appropriate definitions and semantics depending on various needs. Therefore, in formal argumentation, principles are often more technical. The most discussed principles are admissibility, directionality and scc decomposibility, which all have a technical nature. In this paper, from these we study a generalized notion of directionality, taking into account not only the directionality of the attacks, but also of the supports.

The layout of this paper is as follows. In Section 2 we introduce the four kinds of indirect attack corresponding to deductive and necessary interpretation discussed in the literature on bipolar argumentation. In Section 3 we introduce the four atomic reductions corresponding to these four kinds of indirect attack, and two iterated reductions. In Section 4 we introduce the fragment of bipolar argumentation frameworks with evidential support. In Section 5 we introduce the new principles and we give an analysis of which properties are satisfied by which reduction. Section 6 is devoted to the related work and to some concluding remarks.

Indirect Attacks in Bipolar Argumentation framework

This section gives a brief summary of the concept of indirect attack in bipolar argumentation. Dung's argumentation framework (Dung 1995) consists of a set of arguments and a relation between arguments, which is called attack.

Definition 1 (Argumentation framework (Dung 1995)) *An* argumentation framework *(AF) is a tuple $\langle \mathcal{A}, \mathcal{R} \rangle$ where $\mathcal{A}$ is a set of arguments and $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ is a binary attack relation over $\mathcal{A}$.*

An *AF* can be represented as a directed graph, where the nodes represent arguments, and the edges represent the attack relation: Given $a, b \in \mathcal{A}$, $(a,b) \in \mathcal{R}$ stands for a attacks b, noted as $a \to b$.

Definition 2 (Conflict-freeness & Defense (Dung 1995)) *Let $\langle \mathcal{A}, \mathcal{R} \rangle$ be an AF:*

- *$E \subseteq \mathcal{A}$ is* conflict-free *iff $\nexists a, b \in E$ such that $(a,b) \in \mathcal{R}$.*
- *$E \subseteq \mathcal{A}$* defends *c iff $\forall b \in \mathcal{A}$ with $(b,c) \in \mathcal{R}$, $\exists a \in E$ such that $(a,b) \in \mathcal{R}$.*

We distinguish several definitions of extension, each corresponding to an acceptability semantics that formally rules the argument evaluation process.

Definition 3 (Acceptability semantics (Dung 1995)) *Let $\langle \mathcal{A}, \mathcal{R} \rangle$ be an AF:*

- *$E \subseteq \mathcal{A}$ is* admissible *iff it is conflict-free and defends all its elements.*
- *A conflict-free $E \subseteq \mathcal{A}$ is a* complete extension *iff $E = \{a | E \text{ defends } a\}$.*
- *$E \subseteq \mathcal{A}$ is the* grounded extension *iff it is the smallest (for set inclusion) complete extension.*
- *$E \subseteq \mathcal{A}$ is a* preferred extension *iff it is a largest (for set inclusion) complete extension.*
- *$E \subseteq \mathcal{A}$ is a* stable extension *iff it is a preferred extension that defeats all arguments in $\mathcal{A} \backslash E$.*

Example 1 (Four arguments) *The argumentation framework visualized on the left hand side of Figure 1 is defined by $AF = \langle \{a,b,c,d\}, \{(a,b),(b,a),(c,d),(d,c)\} \rangle$. There are four preferred extensions: $\{a,c\}, \{b,c\}, \{a,d\}, \{b,d\}$, and they are all stable extensions.*

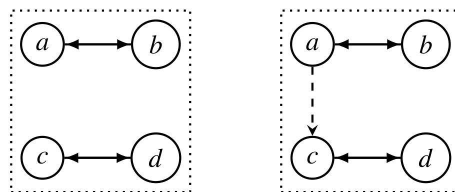


Figure 1: An argumentation framework (AF) and a bipolar argumentation framework (BAF)

Bipolar argumentation framework is an extension of Dung's framework. It is based on a binary attack relation between arguments and a binary support relation over the set of arguments.

Definition 4 (Bipolar argumentation framework) (Cayrol and Lagasquie-Schiex 2005) *A* bipolar argumentation framework *(BAF, for short) is a 3-tuple $\langle \mathcal{A}, \mathcal{R}, \mathcal{S} \rangle$ where $\mathcal{A}$ is a set of arguments, $\mathcal{R}$ is a binary attack relation $\subseteq \mathcal{A} \times \mathcal{A}$ and $\mathcal{S}$ is a binary support relation $\subseteq \mathcal{A} \times \mathcal{A}$, and $\mathcal{R} \cap \mathcal{S} = \phi$. Thus, an AF is a special BAF with the form $\langle \mathcal{A}, \mathcal{R}, \emptyset \rangle$.*

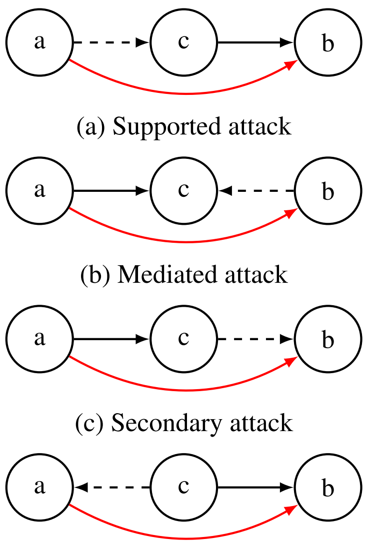
A *BAF* can be represented as a directed graph. Given $a, b, c \in \mathcal{A}$, $(a,b) \in \mathcal{R}$ means a attacks b, noted as $a \to b$; $(b,c) \in \mathcal{S}$ means b supports c, noted as $b \dashrightarrow c$.

Example 2 (Four arguments, continued) *The bipolar argumentation framework visualized at the right hand side of Figure 1 extends the argumentation framework in Example 1 such that a supports c.*

Support relations only influence the extensions when there are also attacks, leads to the study of the interactions between attack and support. In the literature, the different kinds of relations between support and attack have been studied as different notions of indirect attack.

Definition 5 (Four indirect attacks (Polberg 2017)) *Let $BAF = \langle \mathcal{A}, \mathcal{R}, \mathcal{S} \rangle$ be a BAF and $a, b \in \mathcal{A}$, there is:*

- *a supported attack from a to b in BAF iff there exists an argument c s.t. there is a sequence of supports from a to c and c attacks b, represented as* $(a,b) \in \mathcal{R}^{sup}$.
- *a mediated attack from a to b in BAF iff there exists an argument c s.t. there is a sequence of supports from b to c and a attacks c, represented as* $(a,b) \in \mathcal{R}^{med}$.
- *a secondary attack from a to b in BAF iff there exists an argument c s.t. there is a sequence of supports from c to b and a attacks c,* $(a,b) \in \mathcal{R}^{sec}$.
- *a extended attack from a to b in BAF iff there exists an argument c s.t. there is a sequence of supports from c to a and c attacks b,* $(a,b) \in \mathcal{R}^{ext}$.



(a) Supported attack

(b) Mediated attack

(c) Secondary attack

(d) Extended attack

Figure 2: Four kinds of indirect attack

Definition 6 (Super-mediated attack (Cayrol and Lagasquie-Schiex 2013)) *Let* $BAF = \langle \mathcal{A}, \mathcal{R}, \mathcal{S} \rangle$ *be a BAF and* $a, b \in \mathcal{A}$*, there is a super-mediated attack from a to b in BAF iff there exists an argument c s.t. there is a sequence of supports from b to c and a direct or supported attacks c, represented as* $(a,b) \in \mathcal{R}^{med}_{R^{sup}}$.

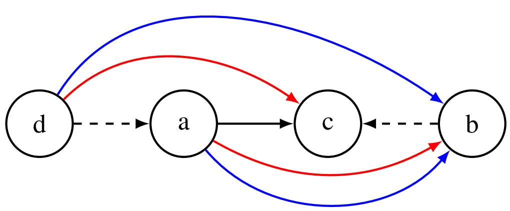


Figure 3: Super-mediated attack

We can obtain various kinds of indirect attacks according to different interpretation of support relation. These indirect attacks were built from the combination of direct attacks and the supports, then from the obtained indirect attacks and the support we can build additional indirect attacks and so on.

Definition 7 (Tiered indirect attacks(Polberg 2017)) *Let* $BF = (A, R, S)$ *be a BAF. The tiered indirect attacks of BF are as follows :*

- ${R_0}^{ind} = \emptyset$
- $R_1^{ind} = \{R_\emptyset^{sup}, R_\emptyset^{sec}, R_\emptyset^{med}, R_\emptyset^{ext}\}$
- $R_i^{ind} = \{R_E^{sup}, R_E^{sec}, R_E^{med}, R_E^{ext} \mid E \subseteq R_{i-1}^{ind}\}$ *for* $i > 1$*, where:*
  - $R_E^{sup} = \{(a,b) \mid$ *there exists an argument c s.t. there is a sequence of supports from a to c and* $(c,b) \in R \cup \bigcup E\}$
  - $R_E^{sec} = \{(a,b) \mid$ *there exists an argument c s.t. there is a sequence of supports from c to b and* $(a,c) \in R \cup \bigcup E\}$
  - $R_E^{med} = \{(a,b) \mid$ *there exists an argument c s.t. there is a sequence of supports from b to c and* $(a,c) \in R \cup \bigcup E\}$
  - $R_E^{ext} = \{(a,b) \mid$ *there exists an argument c s.t. there is a sequence of supports from c to a and* $(c,b) \in R \cup \bigcup E\}$

With R^{ind} *we will denote the collection of all sets of indirect attacks* $\bigcup_{i=0}^{\infty} R_i^{ind}$

Deductive and necessary support

In this section we rephrase the different kinds of indirect attacks as an intermediate step towards semantics for bipolar argumentation frameworks. The reductions can be used together with definitions 2 and 3 to define the extensions of a bipolar argumentation framework.

The notion of conflict-free does not change, in the sense that the conflict-free principle for bipolar frameworks is defined in the same way as the related principle for Dung's theory, though now also indirect attacks are taken into account. For example, support relations can help arguments to defend against other arguments, and in general support relations can influence the acceptability of arguments in various ways. If we have a bipolar argumentation framework without support relations we would like to recover Dung's definitions 2 and 3, such that bipolar argumentation is a proper extension of Dung's argumentation. Moreover, if we have a bipolar argumentation framework without attack relations, then we would like to accept all arguments, for all semantics.

The idea of the atomic reductions is that we interpret all the support relations of the framework according to one of the types of support. This will help us in the analysis of the behavior of the different kinds of support.

Definition 8 (Existing reductions of BAF to AF) *Given a* $BAF = \langle \mathcal{A}, \mathcal{R}, \mathcal{S} \rangle, \forall a, b, c \in \mathcal{A}$*:*

- *SupportedReduction (Cayrol and Lagasquie-Schiex 2013) (RS for short):* $(a,b) \in \mathcal{R}^{sup}$ *is the collection of supported attack iff* $(a,c) \in \mathcal{S}$ *and* $(c,b) \in \mathcal{R}$*,* $RS(BAF) = (\mathcal{A}, \mathcal{R} \cup \mathcal{R}^{sup})$.
- *MediatedReduction (Cayrol and Lagasquie-Schiex 2013) (RM for short):* $(a,b) \in \mathcal{R}^{med}$ *is the collection of mediated attack iff* $(b,c) \in \mathcal{S}$ *and* $(a,c) \in \mathcal{R}$*,* $RM(BAF) = (\mathcal{A}, \mathcal{R} \cup \mathcal{R}^{med})$.

- *SecondaryReduction (Cayrol and Lagasquie-Schiex 2013) (R2 for short):* $(a,b) \in \mathcal{R}^{sec}$ *is the collection of secondary attack iff* $(c,b) \in \mathcal{S}$ *and* $(a,c) \in \mathcal{R}$, $R2(BAF) = (\mathcal{A}, \mathcal{R} \cup \mathcal{R}^{sec})$.
- *ExtendedReduction (Cayrol and Lagasquie-Schiex 2013) (RE for short):* $(a,b) \in \mathcal{R}^{ext}$ *is the collection of extended attack, iff* $(c,a) \in \mathcal{S}$ *and* $(c,b) \in \mathcal{R}$, $RE(BAF) = (\mathcal{A}, \mathcal{R} \cup \mathcal{R}^{ext})$.
- *DeductiveReduction(Polberg 2017)(RD for short) Let* $R' = \{R^{sup}, R^{med}_{R^{sup}}\} \subseteq R^{ind}$ *the collection of supported and super-mediated attacks in BF,* $RD(BAF) = (A, R \cup \bigcup R')$.
- *NecessaryReduction(Polberg 2017)(RN for short) Let* $R' = \{R^{sec}, R^{ext}\} \subseteq R^{ind}$ *the collection of secondary and extended attacks in BF,* $RN(BAF) = (A, R \cup \bigcup R')$.

In general, we write $\mathcal{E}(BAF)$ for the extensions of a BAF, which is characterized by a reduction and a Dung semantics. We write $\mathcal{E}_S(BAF)$ for the extensions of the bipolar framework using Dung semantics S.

Example 3 (Six reductions, continued) *The reduction of the bipolar argumentation framework in Example 2 is visualized in Figure 4. The reductions lead to the following extensions.*

- *After RS, we get the associated AF with the addition of a attacks b, the preferred extensions are: (a, c), (b, c), (b, d);*
- *After RM, we get the associated AF with the addition of d attacks a, the preferred are extensions: (a, c), (b, c), (b, d);*
- *After R2, we get the associated AF with the addition of a attacks d, the preferred are extensions: (a, c), (a, d), (b, d);*
- *After RE, we get the associated AF with the addition of a attacks d, the preferred are extensions: (a, c), (a, d), (b, d).*
- *After RD, we get the associated AF with the addition of a attacks d, the preferred are extensions: (a, c), (b, c), (b, d).*
- *After RN, we get the associated AF with the addition of a attacks d, the preferred are extensions: (a, c), (a, d), (b, d).*

It should be noted that these atomic reductions can be combined in different ways into more complex notions of reductions. For example, it is common practice to add both the indirect attacks of two types, and also the order in which attacks are added can have an impact.

Evidential support

Evidential support is usually defined for a more general bipolar framework in which sets of arguments can attack or support other arguments. To keep our presentation uniform and to compare evidential support to deductive and necessary support, we only consider the fragment of bipolar argumentation frameworks where individual arguments attack or support other arguments. This also simplifies the following definitions.

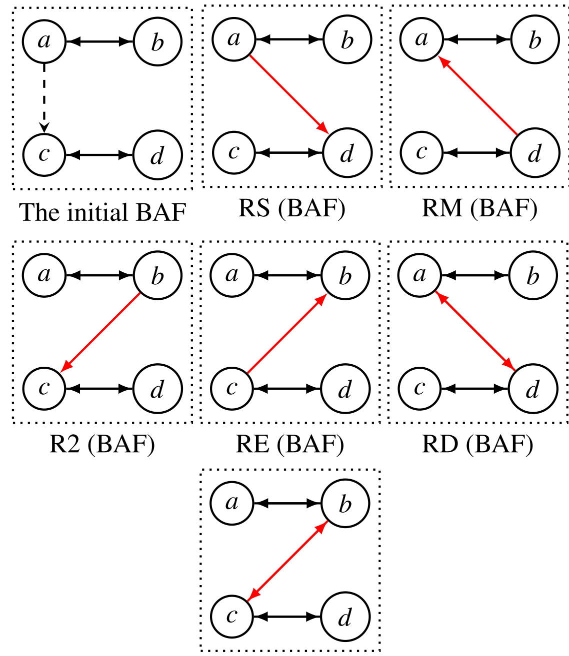


Figure 4: The initial BAF with the associated AFs after Reductions

Moreover, evidential support contains special arguments which do not need to be supported by other arguments. Such arguments may have to satisfy other constraints, for example that they cannot be attacked by ordinary arguments, or that they cannot attack ordinary arguments. To keep our analysis uniform, we do not explicitly distinguish such special arguments, but encode them implicitly: if an argument supports itself, then it is such a special argument. This leads to the following definition of an evidential sequence for an argument.

Definition 9 (Evidential sequence) *Given a BAF* $= \langle \mathcal{A}, \mathcal{R}, \mathcal{S} \rangle$. *A sequence* $(a_0, \ldots, a_n)$ *of elements of* $\mathcal{A}$ *is an evidential sequence for argument* a_n *iff* $(a_0, a_0) \in \mathcal{S}$, *and for* $0 \leq i < n$ *we have* $(a_i, a_{i+1}) \in \mathcal{S}$.

Definition 10 (e-Defense & e-Admissible) *Given a BAF* $= \langle \mathcal{A}, \mathcal{R}, \mathcal{S} \rangle$, *a set of arguments* $S \subseteq \mathcal{A}$ *e-defends argument* $a \in \mathcal{A}$ *iff for every evidential sequence* $(a_0, \ldots, a_n)$ *where* a_n *attacks a, there is an argument* $b \in S$ *attacking one of the arguments of the sequence. Moreover, a set of arguments S is e-admissible iff*

- *for every argument* $a \in S$ *there is an evidential sequence* $(a_0, \ldots, a)$ *such that each* $a_i \in S$ *(a is e-supported by S),*
- *S is conflict free, and*
- *S e-defends all its elements.*

In line with Dung's definitions, a set of arguments is called an e-complete extension if it is e-admissible and it contains all arguments it e-defends; it is e-grounded extension iff it is a minimal e-complete extension; and it is e-preferred if it is maximal e-admissible extension. Moreover, it is e-stable if *for every for every evidential sequence $(a_0, \dots, a_n)$ where a_n not in S, we have an argument $b \in S$ attacking an element of the sequence. We use $REv(BAF)$ to represent the associated AF of the BAF with evidential support.*

The traditional definitions add moreover that elements of evidential support are unique, that support is minimal, and so on. This does not affect the definition of the extensions, and we therefore do not consider that in this paper.

Finally, there is also a kind of reduction of bipolar frameworks to Dung frameworks, but this does not work by a reduction of support relations to attack relations. Instead, it is based on a kind of meta-argumentation, in which the arguments of the Dung framework consists of sets or sequences of arguments in the bipolar framework. As this reduction is not directly relevant for the concerns of this paper, we refer the reader to the relevant literature (Polberg 2017).

Principle-based analysis based on the different kinds of indirect attacks and property

In this section we introduce principles corresponding to the different notions of indirect attack. They correspond to constraints TRA, nATT and n+ATT Cayrol et al. (Cayrol and Lagasquie-Schiex 2015). Basically these properties correspond to the interpretations underlying the different kinds of support.

Principle 1 (Transitivity) *For each $BAF = \langle \mathcal{A}, \mathcal{R}, \mathcal{S} \rangle$, if $a\mathcal{S}b$ and $b\mathcal{S}c$, then $\mathcal{E}\langle \mathcal{A}, \mathcal{R}, \mathcal{S} \rangle = \mathcal{E}\langle \mathcal{A}, \mathcal{R}, \mathcal{S} \cup \{a\mathcal{S}c\} \rangle$.*

Principle 2 (Supported attack) *For each $BAF = \langle \mathcal{A}, \mathcal{R}, \mathcal{S} \rangle$, if $a\mathcal{S}c$ and $c\mathcal{R}b$, then $\mathcal{E}\langle \mathcal{A}, \mathcal{R}, \mathcal{S} \rangle = \mathcal{E}\langle \mathcal{A}, \mathcal{R} \cup \{a\mathcal{R}b\}, \mathcal{S} \rangle$.*

Principle 3 (Mediated attack) *For each $BAF = \langle \mathcal{A}, \mathcal{R}, \mathcal{S} \rangle$, if $b\mathcal{S}c$ and $a\mathcal{R}c$, then $\mathcal{E}\langle \mathcal{A}, \mathcal{R}, \mathcal{S} \rangle = \mathcal{E}\langle \mathcal{A}, \mathcal{R} \cup \{a\mathcal{R}b\}, \mathcal{S} \rangle$.*

Principle 4 (Secondary attack) *For each $BAF = \langle \mathcal{A}, \mathcal{R}, \mathcal{S} \rangle$, if $c\mathcal{S}b$ and $a\mathcal{R}c$, then $\mathcal{E}\langle \mathcal{A}, \mathcal{R}, \mathcal{S} \rangle = \mathcal{E}\langle \mathcal{A}, \mathcal{R} \cup \{a\mathcal{R}b\}, \mathcal{S} \rangle$.*

Principle 5 (Extended attack) *For each $BAF = \langle \mathcal{A}, \mathcal{R}, \mathcal{S} \rangle$, if $c\mathcal{S}a$ and $c\mathcal{R}b$, then $\mathcal{E}\langle \mathcal{A}, \mathcal{R}, \mathcal{S} \rangle = \mathcal{E}\langle \mathcal{A}, \mathcal{R} \cup \{a\mathcal{R}b\}, \mathcal{S} \rangle$.*

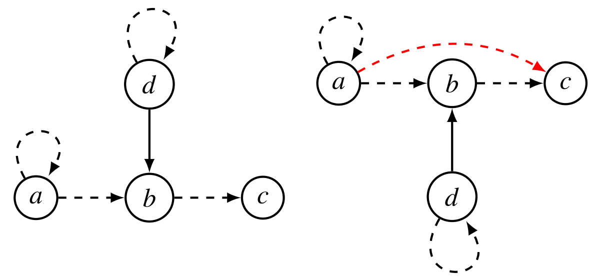
Proposition 1 *REv does not satisfy Principle 1 for all the semantics.*

Proof 1 *We use a counterexample to proof REv does not satisfy Principle 1 for e-complete semantics. Assume a $BAF = \langle \mathcal{A}, \mathcal{R}, \mathcal{S} \rangle$, in which $\mathcal{A} = \{a, b, c, d\}$, $\mathcal{R} = \{(d, b)\}$, $\mathcal{S} = \{(a, a), (a, b), (b, c)(d, d)\}$, the e-complete semantics of BAF is $\{a, d\}$. Because a supports c and c supports b, s.t. a supports c, then we have $BAF' = \langle \{a, b, c, d\}, \{(d, b)\}, \{(a, a), (a, b), (b, c)(d, d), (a, c)\} \rangle$, the e-complete semantics of BAF' is $\{a, c, d\}$, see Figure 5.*

The table below shows the correspondence between the reductions and the first five principles. We omit the straightforward proofs.

Basic principles

We start with the basic property from Baroni's classification (Baroni and Giacomin 2007), conflict-freeness. Since we only add attack relations, and all Dung's semantics satisfy the conflict-free principle, the property of conflict-freeness is trivially satisfied for all reductions.



The initial BAF $\quad$ BAF'

Figure 5: The counterexample of Proof 1

Table 1: Comparison among the reductions and the proposed principles. We refer to Dung's semantics as follows: Complete ($\mathbb{C}$), Grounded ($\mathbb{G}$), Preferred ($\mathbb{P}$), Stable ($\mathbb{S}$). When a principle is never satisfied by a certain reduction for all semantics, we use the $\times$ symbol. P1 refers to Principle 1, the same holds for the others.

Red.	P1	P2	P3	P4	P5
RS	$\mathbb{CGPS}$	$\mathbb{CGPS}$	$\times$	$\times$	$\times$
RM	$\mathbb{CGPS}$	$\times$	$\mathbb{CGPS}$	$\times$	$\times$
R2	$\mathbb{CGPS}$	$\times$	$\times$	$\mathbb{CGPS}$	$\times$
RE	$\mathbb{CGPS}$	$\times$	$\times$	$\times$	$\mathbb{CGPS}$
RD	$\mathbb{CGPS}$	$\mathbb{CGPS}$	$\times$	$\mathbb{CGPS}$	$\times$
RN	$\mathbb{CGPS}$	$\times$	$\mathbb{CGPS}$	$\times$	$\mathbb{CGPS}$
REv	$\times$	$\times$	$\times$	$\times$	$\times$

Principle 6 (Conflict-free) *Given a $BAF = (\mathcal{A}, \mathcal{R}, \mathcal{S})$, if $(a, b) \in \mathcal{R}$, then $\nexists E \in \mathcal{E}(BAF)$ s.t. $(a, b) \in E$.*

The important principle of closure of an extension under supported arguments was introduced by Cayrol et al. (Cayrol and Lagasquie-Schiex 2015), called

Principle 7 (Closure) *Given a $BAF = (\mathcal{A}, \mathcal{R}, \mathcal{S})$, for all extensions E in $\mathcal{E}$, $\forall a, b \in \mathcal{A}$, if $a\mathcal{S}b$ and $a \in E$, then $b \in E$.*

The following propositions show that closure under supported arguments holds only for some reductions.

Proposition 2 *RS and RM satisfy Principle 7 for all the semantics.*

Proof 2 *To prove Proposition 2, we use proof by contradiction. Let $E \subseteq \mathcal{A}$ be a complete extension of an AF which is the associated framework of a BAF after RM. Assume RM does not satisfy Principle 7 for complete semantic, s.t. $\exists a \in E$, $b \in \mathcal{A} \backslash E$, s.t. $(a, b) \in \mathcal{S}$. As $b \notin E$, s.t. $\exists c \in A$, $(c, b) \in R$, but $\nexists d \in E$ s.t. d defends b, i.e., d attacks c. If c attacks b, then c mediated attacks a, there is no d attacks c, then E is not admissible. There is a contradiction between E is not admissible and E is complete. Therefore, RM satisfies Principle 7 for complete semantics.*

Polberg (Polberg 2017) introduces a variant of closure, called inverse closure.

Principle 8 (Inverse Closure) *Given $BAF = \langle \mathcal{A}, \mathcal{R}, \mathcal{S} \rangle$, for all extensions E in $\mathcal{E}$, $\forall a, b \in \mathcal{A}$, if $a\mathcal{S}b$ and $b \in E$, then $a \in E$.*

The following proposition shows that the reductions that do not satisfy closure, satisfy the inverse closure principle instead. Consequently, closure and inverse closure are good principles to distinguish the behavior of the reductions.

Proposition 3 *R2 and RE satisfy Principle 8 for all the semantics.*

Proof 3 *To prove Proposition 3, we use proof by contradiction. Let $E \subseteq A$ be a complete extension of an AF which is the associated framework of a BAF after R2. Assume R2 does not satisfy Principle 8, then $b \in E$, $a \notin E$, s.t. $(a,b) \in \mathcal{S}$. As $a \notin E$, s.t. $\exists c \in A$, $(c,a) \in R$, but $\nexists d \in E$ s.t. d defends a, i.e., d attacks c. If c attacks a, then c secondary attacks b, there is no d attacks c, then E is not admissible. There is a contradiction between E is not admissible and E is complete. Therefore, R2 satisfies Principle 8 for complete semantics.*

Dynamic principles

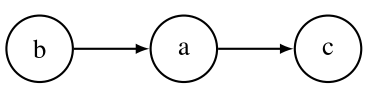
Dynamic properties often give insight in the behavior of semantics. Principle 9 says that adding support relations can only lead to a decrease of extensions, as in Example 3.

Principle 9 (Number of extensions) $|\mathcal{E}_S(\mathcal{A},\mathcal{R},\mathcal{S}\cup\mathcal{S}')| \leqslant |\mathcal{E}_S(\mathcal{A},\mathcal{R},\mathcal{S})|$

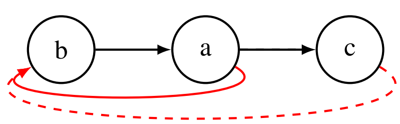
However, Principle 9 only holds for the grounded semantics. Below is the proof for R2 does not satisfy Principle 9 for preferred semantics, we omit other proofs due to lack of space.

Proposition 4 *All the reductions do not satisfy Principle 9 for all the semantics except for grounded semantics.*

Proof 4 *We use a counterexample to prove R2 does not satisfy Principle 9 for preferred semantics. Assume a BAF $= \langle \mathcal{A},\mathcal{R},\mathcal{S}\rangle$, in which $\mathcal{A} = \{a,b,c\}$, $\mathcal{R} = \{(b,a),(a,c)\}$, $\mathcal{S} = \emptyset$, the preferred semantics of BAF is $\{b,c\}$. Let c supports b, then we have $BAF' = \langle\{a,b,c\}, \{(b,a),(a,c)\}, \{(c,b)\}\rangle$, the preferred semantics of BAF' is $\{a\}$ and $\{b,c\}$.*



(a) The initial BAF



(b) BAF': After the addition of support

Figure 6: The counterexample of Proof 4

Proof 5 *We use a counterexample to prove REv does not satisfy Principle 9 for e-complete semantics. Assume a BAF $= \langle \mathcal{A},\mathcal{R},\mathcal{S}\rangle$, in which $\mathcal{A} = \{a,b,c,d\}$, $\mathcal{R} = \{(a,b)(b,a)\}$, $\mathcal{S} = \{(c,c)(c,a)\}$, the e-complete semantics of BAF is $\{a,c\}$. Let d supports d, then we have $BAF' = \langle\{a,b,c,d\}, \{(a,b)(b,a)\}, \{(c,c),(c,a),(d,d)\}\rangle$, the e-complete semantics of BAF' is $\{a,c,d\}$ and $\{b,c,d\}$.*

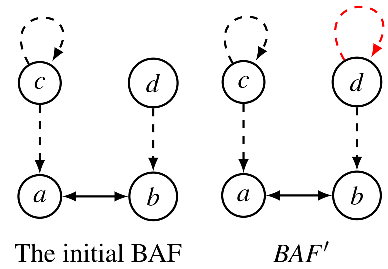


Figure 7: The counterexample of Proof 5

A more fine-grained analysis is based on dynamic properties that consider the addition of relations in certain cases. The following principle considers the addition of support relations among arguments which are both accepted.

Principle 10 (Addition persistence) *Suppose E is an extension of a BAF, and $a,b \in E$. Now BAF' is the framework with the addition of a support relation from a to b, i.e. $\mathcal{E}_S(\mathcal{A},\mathcal{R},\mathcal{S}) = \mathcal{E}_S(\mathcal{A},\mathcal{R},\mathcal{S}\cup(a,b))$. We have that E is also an extension of BAF'.*

As expected, addition persistence holds for all reductions.

Proposition 5 *All the reductions satisfy Principle 10 for all the semantics.*

Proof 6 *Due to the lack of space we only provide proof sketch. While two arguments are already in E which is an extension of a BAF, we add support relation between them, then there are three situations: the first is no new attack needs to be added; the second is a new attack from an argument inside this extension to an outside argument, which has no influence to this extension; the third is the addition of a new attack from the outside argument to an inside argument, the attacked argument is still defended. Thus E is still an extension of BAF.*

Along the same lines, the following principle considers the removal of support relations in specific cases.

Principle 11 (Removal persistence) *Suppose E is an extension of a BAF, $\forall a,b,c \in \mathcal{A}$, where a supports c and c attacks b, $a \in E$ but $b \notin E$. Now BAF' is the framework which removes the support relation from a to c, $\mathcal{E}_S(\mathcal{A},\mathcal{R},\mathcal{S}) = \mathcal{E}_S(\mathcal{A},\mathcal{R},\mathcal{S}\backslash(a,b))$, we have that E is also an extension of BAF'; Or we suppose E is an extension of a BAF, $\forall a,b,c \in \mathcal{A}$, where c supports a and c attacks b, $a \in E$ but $b \notin E$. Now BAF' is the framework which removes the support relation from c to a, $\mathcal{E}_S(\mathcal{A},\mathcal{R},\mathcal{S}) = \mathcal{E}_S(\mathcal{A},\mathcal{R},\mathcal{S}\backslash(a,b))$, We have that E is also an extension of BAF'.*

Principle 11 only holds for some reductions.

Proposition 6 *RM and R2 do not satisfy Principle 11 for the all the semantics.*

Proof 7 *We use a counterexample to prove RM and R2 do not satisfy Principle 11 for preferred semantics. The preferred semantics of Figure 2(b) is $\{a\}$, if we delete the support relation from b to c, the preferred semantics turns to $\{a,b\}$; The preferred semantics of Figure 2(c) is $\{a\}$, if we* *delete the support relation from c to b, the preferred semantics turns to* $\{a,b\}$.

Directionality

Directionality can be generalized to bipolar argumentation as follows.

Definition 11 (Unattacked and unsupported arguments in BAF) *Given an BAF* $= \langle \mathcal{A}, \mathcal{R}, \mathcal{S} \rangle$, *a set U is unattacked and unsupported if and only if there exists no* $a \in \mathcal{A} \backslash U$ *such that a attacks U or a supports U. The unattacked and unsupported sets in BAF is denoted* $US(BAF)$ *(U for short).*

Principle 12 (BAF Directionality) *A BAF semantics* σ *satisfies the BAF directionality principle iff for every BAF, for every* $U \in US(BAF)$, *it holds that* $\sigma\ (BAF_{\downarrow U}) = \{E \cap U | E \in \sigma(BAF)\}$, *where for* $BAF = \langle \mathcal{A}, \mathcal{R}, \mathcal{S} \rangle$, $BAF_{\downarrow U} = (U, \mathcal{R} \cap U \times U, \mathcal{S} \cap U \times U)$ *is a projection, and* $\sigma\ (BAF_{\downarrow U})$ *are the extensions of the projection.*

In (Baroni and Giacomin 2007), the authors have showed that stable semantics violates directionality, here we omit the proof of all the reductions do not satisfy Principle 12 for stable semantics.

Proposition 7 *RS satisfies Principle 12 for grounded, complete and preferred semantics.*

Proof 8 *To prove Proposition 7, we use proof by contradiction. Assume RS does not satisfy Principle 12, Let* U_1 *be an unattacked and unsupported set, let* U_2 *be* $\mathcal{A} \backslash U$, *we assume a semantics for AF that satisfies Principle 12 for grounded, complete and preferred semantics. From the above, we have a supports c and c attacks b, s.t. a supported attacks b, a is in* U_2 *and b is in* U_1. *If b is in* U_1, *then c must be in* U_1, *if c is in* U_1, *then a must be in* U_1. *Contradiction.*

Proposition 8 *R2 satisfies Principle 12 for grounded, complete and preferred semantics..*

Proof 9 *To prove Proposition 8, we use proof by contradiction. Assume R2 does not satisfy Principle 12, Let* U_1 *be an unattacked and unsupported set, let* U_2 *be* $\mathcal{A} \backslash U$, *we assume a semantics for AF that satisfies Principle 12 for grounded, complete and preferred semantics. From the above, we have a supports c, c attacks b, s.t. a secondary attacks b, a is in* U_2 *and b is in* U_1. *If b is in* U_1, *then c must be in* U_1, *if c is in* U_1, *then a must be in* U_1. *Contradiction.*

Proposition 9 *RE does not satisfy Principle 12 for grounded, complete and preferred semantics.*

Proof 10 *We use a counterexample to prove RE does not satisfy Principle 12 for preferred semantics. Assume we have a BAF visualized as the left in Figure 6, argument c supports a, then we have the associated AF visualized as the middle in Figure 8 in which we add a extended attacks b and the same form a to d. From the initial BAF, we have an unattacked and unsupported set* $U = \{b,c,d\}$, *the right of Figure 8 visualizes* $BAF_{\downarrow U}$. *The preferred extensions of BAF is* $\sigma(BAF) = \sigma(AF) = \{\{a,c\}\}$, $\sigma\ (BAF_{\downarrow U}) = \{\{c\},\{b,d\}\}$, $\sigma(BAF) \cap U = \{\{c\}\}$, $\sigma\ (BAF_{\downarrow U}) \neq \{\{c\}\}$. *Thus,* $BAF_{\downarrow U}$ *is not the projection of whole framework.*

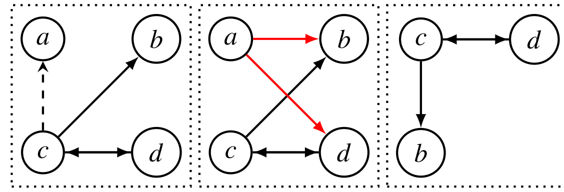


Figure 8: The counterexample for Proof 10

Proposition 10 *RM does not satisfy Principle 12 for grounded, complete and preferred semantics.*

Proof 11 *We use a counterexample to prove RM does not satisfy Principle 12 for preferred semantics, which is showed in Figure 9. Due to the lack of space, here we omit the details.*

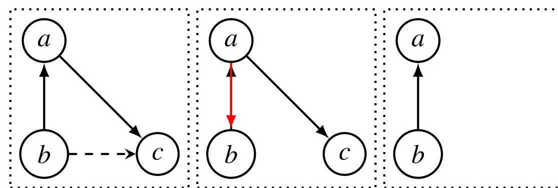


Figure 9: The counterexample for Proof 11

Supported arguments

Principle 13 (Global support) *Given a BAF* $= \langle \mathcal{A}, \mathcal{R}, \mathcal{S} \rangle$, *for all extensions E in* $\mathcal{E}$, *if* $a \in E$, *then there must be an argument b s.t.* $b \in E$, *and b supports a.*

Principle 14 (Grounded support) *Given a BAF* $= \langle \mathcal{A}, \mathcal{R}, \mathcal{S} \rangle$, *for all extensions E in* $\mathcal{E}$, *if* $a \in E$, *then there must be an argument* $b \in E$ *and* $(b,b) \in \mathcal{S}$ *(or* $(a,a) \in \mathcal{S}$*), s.t. there is a support sequence* $(b, a_0, \ldots, a_n, a)$, *all* $a_i \in E$.

Proposition 11 *All the reductions except for REv do not satisfy Principle 13 and 14 for all the semantics.*

Proof 12 *We can simply use a counterexample to prove Proposition 11. Assume we have a BAF* $= \langle \{a\}, \emptyset, \emptyset \rangle$, $\mathcal{E}_{complete}(RS(BAF))$=$\mathcal{E}_{complete}(RM(BAF)) = \mathcal{E}_{complete}(R2\ (BAF)) = \mathcal{E}_{complete}(RE(BAF)) = \mathcal{E}_{complete}(RD(BAF)) = \mathcal{E}_{complete}\ (RN(BAF)) = \{\{a\}\}$. *However, there is no argument supports a.*

The following table summarizes the results of this section.

Concluding remarks, related and future work

Actually, there is a gap between the formal analysis of bipolar frameworks, i.e.knowledge reasoning, and the informal representation, i.e.knowledge representation. In (Cayrol and Lagasquie-Schiex 2013) , the authors give the following example written in natural language: "a bipolar degree supports a scholarship". The interpretation of this sentence is subjective. You can whether give the support necessary interpretation: "A bachelor degree is necessary for a scholarship, so if someone does not have a bachelor degree, one

Table 2: Comparison among the reductions and the proposed principles.

Red.	P6	P7	P8	P9	P10	P11	P12	P13	p14
RS	$\mathbb{CGPS}$	$\mathbb{GCPS}$	$\times$	$\mathbb{G}$	$\mathbb{CGPS}$	$\mathbb{GCPS}$	$\mathbb{CGP}$	$\times$	$\times$
RM	$\mathbb{CGPS}$	$\mathbb{GCPS}$	$\times$	$\mathbb{G}$	$\mathbb{CGPS}$	$\times$	$\times$	$\times$	$\times$
R2	$\mathbb{CGPS}$	$\times$	$\mathbb{GCPS}$	$\mathbb{G}$	$\mathbb{CGPS}$	$\times$	$\mathbb{CGP}$	$\times$	$\times$
RE	$\mathbb{CGPS}$	$\times$	$\mathbb{GCPS}$	$\mathbb{G}$	$\mathbb{CGPS}$	$\mathbb{GCPS}$	$\times$	$\times$	$\times$
RD	$\mathbb{CGPS}$	$\mathbb{GCPS}$	$\times$	$\mathbb{G}$	$\mathbb{CGPS}$	$\times$	$\times$	$\times$	$\times$
RN	$\mathbb{CGPS}$	$\times$	$\mathbb{GCPS}$	$\mathbb{G}$	$\mathbb{CGPS}$	$\times$	$\times$	$\times$	$\times$
REv	$\mathbb{CGPS}$	$\times$	$\times$	$\mathbb{G}$	$\mathbb{CGPS}$	$\times$	$\times$	$\mathbb{GCPS}$	$\mathbb{GCPS}$

does not get a scholarship"; or give a deductive interpretation: "A bachelor degree is sufficient for a scholarship, so if one does not get a scholarship, one does not have a bachelor degree". This translation from natural language to formal one is standard pragmatics, i.e. whether "A supports B" means "A implies B" (sufficient reason) or "B implies A" (necessary reason), or to mix them to get a more complicated relation. As a result, different agents have different interpretations, formal argumentation may play the meta-dialogue to settle this issue such as we can adopt it for legal interpretation.

However, the considerations above do not invalidate our work about the principle-based approach for bipolar argumentation, on the contrary, because of the ambiguity at the pragmatic and semantic level, a principle-based approach can be very useful to better understand the choices of a particular formalization.

In this paper, we have proposed an axiomatic approach to bipolar argumentation framework semantics, which is summarized in tables 1 and 2 of this paper. We considered seven reductions from bipolar argumentation frameworks to a Dung-like abstract argumentation, four standard semantics to compute the set of accepted arguments, and fourteen principles to study the considered reductions. This work can be extended by considering more reductions, more semantics, and more principles. Our principles are all independent of which admissibility-based semantics is used, though some principles do not hold for semi-stable semantics. Moreover, they do not hold for some of the naive-based semantics.

Some general insights can be extracted from the tables. Our principles P6, P11 and P12 can be used to distinguish among different kinds of reductions, and can be used to choose a reduction for a particular application. Principles like P9 which never hold can be used in the further search for semantics. Also we can define new semantics directly associating extensions with bipolar argumentation frameworks, i.e. without using a reduction.

The results of this paper give rise to many new research questions. We intend to analyze the similarity between reductions for preference-based argumentation frameworks and for bipolar argumentation frameworks. Whereas in both frameworks, the support relation and the preference can be both added and removed. In this way, the theory of reductions for preference based argumentation and bipolar argumentation is closely related to dynamic principles for AF (Rienstra, Sakama, and van der Torre 2015), which can be a source of further principles. Similarly, like in preference-based argumentation, symmetric attack can be studied.

Furthermore, the first volume of the handbook of formal argumentation (Baroni, Gabbay, and Giacomin 2018)surveys the definitions, computation and analysis of abstract argumentation semantics depending on different criteria to decide the sets of acceptable arguments, and various extensions of Dung's framework have been proposed. There are many topics where bipolar argumentation could be used, and such uses could inspire new principles. Gordon (Gordon 2018) requirements analysis for formal argumentation suggests that attack and support should be treated as equals in formal argumentation, which is also suggested by applications like DebateGraph. The handbook discusses also many topics where the theory of bipolar argumentation needs to be further developed. A structured theory of argumentation seems to be needed most. For example, maybe the most natural kind of support is a lemma supporting a proof. This corresponds to the idea of a sub-argument supporting its super-arguments. In Toulmin's argument structure, support arguments could be used as a warrant. Moreover, the role of support in dialogue needs to be clarified. Prakken argues that besides argumentation as inference, there is also argumentation as dialogue, several chapters of the handbook are concerned with this, such as argumentation schemes. The core of the theory is a set of critical questions, which can be interpreted as attacks. Maybe the answers to the critical questions can be modeled as support?

Finally, like Doutre et al (P. et al. 2017), we believe that the scope of the "principle-based approach" of argumentation semantics (Baroni and Giacomin 2007) can be widened. In the manifesto (Gabbay et al. 2018), it is argued that axioms are a way to relate formal argumentation to other areas of reasoning, e.g. social choice.

Acknowledgement

This project has received funding from the European Union's Horizon 2020 research and innovation program under the Marie Skłodowska-Curie ITN EJD grant agreement No 814177.
We acknowledge Dr.Srdjan Vesic and Dr.Tjitze Rienstra for giving valuable advice.

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